Math 201 Test 2, Spring 2012

No calculators and no notes are allowed on this exam. Total time: 1 hour.

For all problems except multiple choice ones and #6, show appropriate work to receive credits and circle/box your answer if possible. When you are reducing a matrix, please indicate the row operations used (in the form \(-2R_1 + R_2 \rightarrow R_2\)).

For questions from #1 to #5, just circle the letter corresponding to the correct answer. Work will not be graded.

1. (6pts) Determine the domain of \( g(x) = 2 + \log_3(x-5) \).
   a. \((-\infty, \infty)\)
   b. \((-\infty, 5]\)
   c. \((5, \infty)\)
   d. \((-\infty, 5)\)

2. (6pts) In order to have no solutions, two lines must always have:
   a. The same slope and the same \(y\)-intercept
   b. Different slopes and the same \(y\)-intercept
   c. The same slope and different \(y\)-intercepts
   d. Different slopes and different \(y\)-intercepts

3. (6pts) The inverse matrix of \( \begin{bmatrix} 2 & -4 \\ -2 & 5 \end{bmatrix} \) is:
   a. \( \begin{bmatrix} -2 & 4 \\ 2 & -5 \end{bmatrix} \)
   b. \( \begin{bmatrix} 1/2 & -1/4 \\ -1/2 & 1/5 \end{bmatrix} \)
   c. \( \begin{bmatrix} 5 & 4 \\ 2 & 2 \end{bmatrix} \)
   d. \( \begin{bmatrix} 5/2 & 2 \\ 1 & 1 \end{bmatrix} \)
4. (6pts) Given the augmented matrix shown below, what can be concluded about the system of equations?

\[
\begin{bmatrix}
1 & 0 & 0 & 5 \\
0 & 1 & 0 & -2 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\(X_1 = 5\)  \(X_2 = -2\)  \(X_3 = 0\)

(a) There is exactly one solution.
(b) There are no solutions.
(c) There are an infinite number of solutions.
(d) The number of solutions cannot be determined.

5. (6pts) Choose the graph of \(f(x) = 4^{x-2} + 3\).

- [ ] a.
- [ ] b.
- [ ] c.
- [ ] d.
6. (3pts each part) Evaluate.
   a. \( \log_8 8 = 1 \)
   b. \( \log_7 1 = 0 \)
   c. \( \log_4 4^{-3} = -3 \)

7. Solve for \( x \):
   a. (6pts) \( \log_b 3^{-2} = -2 \).
      \[ b^{-2} = 3^{-2} \]
      \[ b = 3 \]
   
   b. (10pts) \( 9^{3x+x^2} = 9^{-2} \).
      \[ 3x + x^2 = -2 \]
      \[ x^2 + 3x + 2 = 0 \]
      \[ (x+2)(x+1) = 0 \]
      \[ x = -1, -2 \]

   c. (10pts) \( \log_{10} x + \log_{10} (x-5) = \log_{10} 14 \).
      \[ \log_{10} (x^2-5x) = \log_{10} 14 \]
      \[ x^2 - 5x = 14 \]
      \[ x^2 - 5x - 14 = 0 \]
      \[ (x-7)(x+2) = 0 \]
      \[ x = -2, 7 \]
      
      So \( x = 7 \)

      Test -2:
      \[ \log_{10}(-2) + \log_{10}(-7) \neq \log_{10}(14) \]
      Excluded

      Test 7:
      \[ \log_{10}(7) + \log_{10}(2) = \log_{10}(14) \]
      \[ \sqrt{ } \]
8. (12pts) Solve the following system of linear equations by using Gauss-Jordan elimination. If the system has infinitely many solutions, write your answer in parameters; if the system has no solutions, write your answer as 'no solutions'.

\[
\begin{align*}
  x_1 + 2x_2 - 2x_3 &= -1 \\
  3x_2 - 6x_3 &= 1 \\
  -1x_2 + 2x_3 &= -\frac{1}{3}
\end{align*}
\]

\[
\begin{bmatrix}
  1 & 2 & -2 \\
  0 & 3 & -6 \\
  0 & -1 & 2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  1 & 2 & -2 \\
  0 & 3 & -6 \\
  0 & 1 & 2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  1 & 2 & -2 \\
  0 & 3 & -6 \\
  0 & 0 & -1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  1 & 2 & -2 \\
  0 & 3 & -6 \\
  0 & 0 & 1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  1 & 2 & -2 \\
  0 & 3 & -6 \\
  0 & 0 & 1
\end{bmatrix}
\]

So:

\[
\begin{align*}
  x_1 &= -2t - \frac{5}{3} \\
  x_2 &= 2t + \frac{1}{3} \\
  x_3 &= t
\end{align*}
\]
9. (4pts each part) Perform the indicated operation, if possible. If it is not possible, write “not possible.”

a. \[
\begin{bmatrix}
2 & 9 \\
-3 & 5
\end{bmatrix}
\begin{bmatrix}
1 \\
8
\end{bmatrix}
\]
no possible (dimensions do not match)

b. \[
\begin{bmatrix}
2 & 1 & -1 \\
-3 & 0 & 5
\end{bmatrix}
\begin{bmatrix}
0 & 3 \\
-5 & 0 \\
-1 & 1
\end{bmatrix}
\]
2x3 3x2
\[
\begin{bmatrix}
6+6+1 \\
6+6+1
\end{bmatrix}
\begin{bmatrix}
0+0+5 \\
0+0+5
\end{bmatrix}
\]
= \[
\begin{bmatrix}
-4 & 5 \\
-5 & -4
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & -2 \\
3 & -6 \\
-2 & 4
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & -2 \\
3 & -6 \\
-2 & 4
\end{bmatrix}
\]

10. The WSU track team needs to rent vans to transport the team to a track meet. The team wants to rent exactly 18 vehicles to transport 165 people. They can rent small vans that seat 7 people each, medium vans that seat 10 people each, or large vans that seat 14 people each. How many vans of each type should the track team rent to meet the specified requirements?

a) (3pts) Define the variables that you are using (what does each variable represent?).

\[ x_1 = \text{# of small vans} \]
\[ x_2 = \text{# of medium vans} \]
\[ x_3 = \text{# of large vans} \]

b) (5pts) Write a system of linear equations to describe the situation.

\[ x_1 + x_2 + x_3 = 18 \]
\[ 7x_1 + 10x_2 + 14x_3 = 165 \]

c) (3pts) Write your system of linear equations in an augmented matrix. You do not have to solve it.

\[
\begin{bmatrix}
1 & 1 & 1 & | & 18 \\
7 & 10 & 14 & | & 165
\end{bmatrix}
\]