1. (10) Let \( A = \begin{bmatrix} 2 & 0 & -4 & 2 & -1 & -4 \\ 1 & 0 & -2 & 1 & 2 & 1 \\ 3 & 1 & -4 & 1 & 1 & 3 \\ -2 & 0 & 4 & -2 & -3 & -1 \\ 1 & 0 & -2 & 1 & 1 & 2 \end{bmatrix} \). Then \( A \) row reduces to \( \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \).

(a) Find a basis for \( \text{Col} \ A \).

(b) Find a basis for \( \text{Nul} \ A \).

(c) What is \( \dim \text{Nul} \ A \)? Explain.

(d) What is \( \text{rank} \ A \)? Explain.

2. (10) Find all values of \( h \) so that the set of vectors \( \left\{ \begin{bmatrix} 4 \\ 4 \\ 2 \\ h \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 6 \end{bmatrix} \right\} \) forms a basis for \( \mathbb{R}^3 \). Justify your answer.

3. (10) Let \( A = \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 0 & -2 & 2 \\ 1 & 3 & 4 & 0 \end{bmatrix} \). Find \( \det(A) \).
4. (10) 

Let \( A = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 4 & 0 \\ 2 & 1 & 4 \end{bmatrix} \).

(a) Find the characteristic polynomial of \( A \). You may leave your answer in factored form.
(b) Find the eigenvalues of \( A \). NOTE: The eigenvalues are integers between zero and ten.

5. (10) 

Let \( B = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{bmatrix} \).

Find a basis for the eigenspace of \( B \) associated with the eigenvalue \( \lambda = 2 \).

6. (20) Answer each of the following questions with justification.

(a) If \( A \) is a 5 \( \times \) 6 matrix, can the columns of \( A \) form a basis for \( \mathbb{R}^5 \)?
(b) If \( A, B, \) and \( C \) are \( n \times n \) matrices, \( A \) is invertible, and \( AB = AC \), then must \( B = C \)?
(c) If \( \mathbf{x} \) is an eigenvector of the 4 \( \times \) 4 matrix \( A \) corresponding to the eigenvalue \( \lambda = 0 \), do the columns of \( A \) span \( \mathbb{R}^4 \)?
(d) If \( A \) is a 3 \( \times \) 4 matrix, what is the largest value of the rank of \( A \)? What is the largest value of the dimension of the null space of \( A \)?
(e) Let \( A \) be a 4 \( \times \) 4 matrix with \( \det(A) = 6 \), and the matrix \( B \) is formed from \( A \) by first interchanging Rows two and three, and then dividing Row one by 2. What is \( \det(B) \)?

7. (5) Construct a 3 \( \times \) 3 triangular matrix \( A \) so that the vector \( \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \) is in \( \text{Col}(A) \).

8. (5) Let \( A = \begin{bmatrix} 3 & 2 & -3 \\ 2 & 0 & 0 \\ 5 & -2 & -1 \end{bmatrix} \). Is \( \mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \) an eigenvector of \( A \)? Justify your answer.

9. (5) Let \( A \) and \( B \) be 3 \( \times \) 3 matrices such that \( \det(A) = 2 \) and \( \det(B) = -3 \). Find each of the following determinants, or indicate that the determinant cannot be found from the information given.

(a) \( \det(B^3) \)
(b) \( \det(3B) \)
(c) \( \det(B^{-1}AB) \)
(d) \( \det(A + B) \)
(e) \( \det(A^{-2}) \)

10. (5) Let \( \lambda \) be an eigenvalue of the \( n \times n \) matrix \( A \). Let \( B = A - \lambda I \). Show that \( B \) is not an invertible matrix.