Concept: History of Math - Zero

In this lesson we will learn the history of zero.

Who invented zero?

When did people start using zero?

Who came up with a symbol for zero?

There was a time when people did not know of the concept zero. They were able to function without the need of a zero. At that time, people knew how to count and do simple manipulations with natural numbers. So, one might wonder how they were able to distinguish numbers, such as

216, 2016 and 2160 without a zero.

It seems that they were perfectly capable of distinguishing numbers based on the usage and descriptions of any given numbers.

For example, let us say a friend asks you, "How much did you pay for that round-trip ticket to England?"

You answer, "Seven ninety nine."

Also, that friend asks, "How much is that medium pepperoni pizza?"

You reply, "Seven ninety nine."
So, based on the context of the questions, your friend will gather that you ticket cost $799 and the pizza $7.99 although your answer was "seven ninety nine" in both cases.
Empty Place Indicator

216  2016  2160

In the early days, as the number system started to evolve, there was a need for a symbol to indicate the empty place. Without an empty place indicator, it would be difficult to distinguish, for example, the numbers 216, 2016 and 2160.

However, the earliest symbol that was used for zero was not 0. It was the wedge symbol "\('.\) The wedge symbol is known to have been used by Babylonians in 400 B.C. Using this symbol one writes two thousand, one hundred and sixty as 216".

Later, in 130 A.D., the Greeks introduced the symbol 0 for zero. Apparently, in Greek "ouden" means nothing and the symbol they picked for zero is simply the first letter of the word "ouden". Still, the only use of zero was an empty place indicator.
Ancient literature shows that in India, either the symbol ० or the word "kha" was used for zero around 200 A.D. Again, the usage was to indicate the empty place. It was the Indian mathematician Brahmagupta (628 A.D.) who used zero as number for the first time.

Brahmagupta was the originator of the concept of negative numbers, and he needed a number called "zero" for developing his mathematical ideas. His writings show the rules that he developed with regard to operations with numbers that are positive, negative or zero. Brahmagupta's rules for addition are:

- The sum of a negative number and zero is negative.
- The sum of a positive number and zero is positive.
- The sum of zero and zero is zero.

His rules for division are,

- Positive divided by positive, or negative by negative, is positive.
- Closer (zero) divided by clipor is nought.
- Positive divided by negative is negative.
- Negative divided by positive is negative.
Bhaskara Period

Following in the first steps of Brahmagupta, the Indian mathematician Bhaskara (1114-1185 A.D.) seems to have worked extensively with the number zero. In one of his writings it said,

"In this quantity consisting of that which has cipher for its divisor, there is no alteration, though many may be inserted or extracted; as no change takes place in the infinite."

It is clear that Bhaskara knew that

$$\frac{a}{0} = \infty$$

and

$$\infty + k = \infty$$

In ancient Indian mathematics work, one also finds the formula

$$\frac{a}{0} * 0 = a$$

If one looks at this equation as

$$\lim_{\varepsilon \to 0} \frac{a}{\varepsilon} * \varepsilon = a$$

then the formula is correct. So, did the early Indian mathematicians know about the concept "infinitesimal"? Maybe.

For example, the following problems with answers were found in Bhaskara era writings:

**Problem 1**

Solve:

$$3 \left( \frac{-2}{0} + \frac{-2}{0} \right) = 63$$

Answer: $x=14$

**Problem 2**

Solve:

$$\left( \frac{y}{0} + y - 9 \right)^2 + \left( \frac{y}{0} + y - 9 \right) * 0 = 90$$

Answer: $x=9$

**Problem 3**

Solve:

$$\left[ \left( \frac{y}{2} + \frac{y}{2} \right) * 0 \right]^2 + 2 \left[ \left( \frac{y}{2} + \frac{y}{2} \right) * 0 \right] = 15$$

Answer: $x=2$

Are the answers to the problems given above correct?

Use the idea of infinitesimal $\varepsilon$ to check and determine the assumption that has been made.

**Click Here**

Problem 1, Problem 2, Problem 3

It should be noted that as recently as 1828, some European researchers believed that $\frac{a}{0} * 0 = 0$, if $a$ is not zero.

Brahmagupta Exercise 2
Problem 1

Let us see how the Indian mathematicians would have arrived at their answers

\[
3 \left( \frac{\nu * 0 + \nu * 0}{2} \right) = 6.3
\]  

(1)

Using the idea that \( \frac{\alpha}{0} * 0 = \alpha \), equation (1) can be reduced to

\[
3(\nu + \frac{\nu}{2}) = 63
\]

\[
\frac{9\nu}{2} = 63
\]

\[
\nu = 14
\]

Now, let us employ the "limit concept" and do the problem again.

Let's re-write equation (1) as,

\[
\lim_{\epsilon \to 0} 3 \left( \frac{\nu * \epsilon + \nu * \epsilon}{\epsilon} \right) = 63
\]

\[
\lim_{\epsilon \to 0} 3 \left( \frac{\nu}{\epsilon} \right) * \epsilon + 3 \left( \frac{\nu/2}{\epsilon} \right) * \epsilon = 63
\]

\[
\lim_{\epsilon \to 0} \frac{\alpha}{\epsilon} * \epsilon = \alpha
\]

Using \( 3\nu + 3\nu/2 = 63 \)

\[
\nu = 14
\]

i.e., \( \nu = 14 \), same as the answer obtained by the Indian mathematicians.
Problem 2

\[
\left[ \left( \frac{y}{\epsilon} + y - 9 \right)^2 + \left( \frac{y}{\epsilon} + y - 9 \right) \right] * 0 = 90
\]

(2)

Expanding we obtain

\[
\left[ \frac{y^2}{\epsilon^2} + y^2 + 81 + \frac{2y^2}{\epsilon} - \frac{18y}{\epsilon} - 18y + \frac{y}{\epsilon} + y - 9 \right] * 0 = 90
\]

(2a)

Again using the idea that \( \frac{a}{0} * 0 = a \) and \( a * 0 = 0 \) equation (2a) can be simplified to

\[
\left[ y^2 + 2y^2 - 18y + y \right] = 90
\]

\[
3y^2 - 17y - 90 = 0
\]

So,

\[
y = \frac{17 \pm \sqrt{17^2 + 4*3*90}}{2*3} = 17 \pm \frac{37}{6}
\]

\[
y = \frac{54}{6} \quad \text{or} \quad y = \frac{-20}{6}
\]

i.e., \( y = \frac{54}{6} \) or \( y = \frac{-20}{6} \)

Simplification gives \( y = 9 \) or \( y = \frac{-10}{3} \)

Now, let us employ the "limit concept" and do the problem again.

Let's re-write equation (2) as,

\[
\lim_{\epsilon \to 0} \left[ \left( \frac{y}{\epsilon} + y - 9 \right)^2 + \left( \frac{y}{\epsilon} + y - 9 \right) \right] * \epsilon = 90
\]

(2b)

Now expanding equation (2b) we obtain,

\[
\lim_{\epsilon \to 0} \left[ \frac{y^2}{\epsilon^2} + y^2 + 81 + \frac{2y^2}{\epsilon} - \frac{18y}{\epsilon} - 18y + \frac{y}{\epsilon} + y - 9 \right] * \epsilon = 90
\]

\[
\lim_{\epsilon \to 0} \left( \frac{y^2}{\epsilon} \right) * \epsilon + \lim_{\epsilon \to 0} \left( y^2 + 81 \right) * \epsilon + \lim_{\epsilon \to 0} \left( \frac{2y^2 - 18y}{\epsilon} \right) * \epsilon
\]

\[
+ \lim_{\epsilon \to 0} \left( 18y + y - 9 \right) * \epsilon + \lim_{\epsilon \to 0} \left( \frac{y}{\epsilon} \right) * \epsilon = 90
\]

\[
\lim_{\epsilon \to 0} \left( \frac{y^2}{\epsilon} \right) + 0 + \left( 2y^2 - 18y \right) + y = 90
\]

\[
\lim_{\epsilon \to 0} \left( \frac{y^2}{\epsilon} \right) + \left( 2y^2 - 17y \right) = 90
\]

(2c)

Since \( \frac{y^2}{\epsilon} \to \infty \) as \( \epsilon \to 0 \), equation (2c) is not meaningful!

So, the answer obtained by the Indian mathematicians is not correct. It is clear that they failed to differentiate between a small number (\( \epsilon \)) and the square of a small number (\( \epsilon^2 \)) in their calculations.
Problem 3

\[
\left[ \left\{ \left( \frac{\alpha}{2} \right) \star 0 \right\}^2 + 2 \left\{ \left( \frac{\alpha}{2} \right) \star 0 \right\} \right] \div 0 = 15 \quad (3)
\]

Expanding we get,

\[
\left[ \left\{ \left( \frac{\alpha^2}{2} + \frac{\alpha^2}{4} \right) \star 0 \right\} + 2 \left\{ \left( \frac{\alpha}{2} \right) \star 0 \right\} \right] \div 0 = 15 \quad (3a)
\]

Use the idea \( \frac{\alpha}{0} \times 0 = \alpha \) to reduce equation (3a) to

\[
\alpha^2 + \frac{2\alpha^2}{2} + \frac{\alpha^2}{4} + 2 \left( \frac{\alpha}{2} \right) = 15
\]

\[
\frac{9\alpha^2}{4} + \frac{6\alpha}{2} = 15
\]

\[
9\alpha^2 + 12\alpha - 60 = 0
\]

\[
3\alpha^2 + 4\alpha - 20 = 0
\]

\[
(3\alpha + 10)(\alpha - 2) = 0
\]

So, \( \alpha = 2 \) or \( \alpha = -\frac{10}{3} \)

As an exercise, try to solve this problem using the "limit concept."
Bhaskara (1114-1185 A.D.)

The square root of half the number of bees in a swarm,

Has flown out upon a jasmine bush;

Eight ninths of the swarm has remained behind;

A female bee flies about a male who is buzzing inside a lotus flower;

In the night, attracted by the flower's sweet odor, he went inside it,

And now he is trapped!

Tell me, most enchanting lady, the number of bees.
Bhaskara (1114-1185 A.D.)

\[ x^2 - 61y^2 = 1 \]

Smallest Positive Integer Solution

\[ x = 1,766,319,049 \]

\[ y = 226,153,980 \]