§2.3 5, 6, 12, 16, 24, 27

MATH 220-01
ASSIGNMENT #6

5. The matrix row-reduces to
$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & -5 \\ 0 & 0 & 0 \end{pmatrix}$$
so that there is no pivot in row 3 $\Rightarrow$ A is not invertible, by IMT (c).

6. The matrix reduces to $\begin{pmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ 0 & 0 & 0 \end{pmatrix}$ so that row 3 does not have a pivot $\Rightarrow$ A is not invertible (IMT (c)).

12. a. <u>true:</u> by IMT : (k) true $\Rightarrow$ (j) true.

b. <u>true:</u> (e) true $\Rightarrow$ (h) true.

c. <u>true:</u> see remark following proof of theorem 8.

d. <u>false:</u> not every linear transformation determined by a matrix and mapping $\mathbb{R}^n$ into $\mathbb{R}^n$ has n pivot positions.

22. true: $Ax = b$ inconsistent for some b
$\Rightarrow$ (g) is false $\Rightarrow$ (f) is false.

16. No: If the columns of the matrix do not span $\mathbb{R}^5$, then they are linearly dependent $\Rightarrow$ the matrix will be singular.

24. The equation $Lx = 0$ always has trivial solution. This fact gives no information about the columns of L.

27. Let W be the inverse of AB. Then $ABW = I$ and $A(BW) = I$. $\Rightarrow$ by (k) of IMT there is D s.t.
$DA = I$.

26. repeat the steps in # 24 to get: ③

basis for Col A: $\left\{ \begin{pmatrix} 3 \\ -2 \\ -5 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 9 \\ 6 \end{pmatrix}, \begin{pmatrix} 5 \\ 7 \\ 3 \\ 3 \end{pmatrix} \right\}$.

basis for $N(A)$: $\left\{ \begin{pmatrix} -3 \\ -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -2.5 \\ -1.5 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\}$.

6. we follow exercise 4:

$$[\,v_1\ v_2\ v_3 \mid u\,] = \left(\begin{array}{ccc|c} 1 & 4 & 5 & -4 \\ -2 & -7 & -8 & 10 \\ 4 & 9 & 6 & -7 \\ 3 & 7 & 5 & -5 \end{array}\right) \xrightarrow[R_4 - 3R_1]{R_2 + 2R_1,\ R_3 - 4R_1}$$

$$\xrightarrow[R_4 + 5R_2]{R_1 - 4R_2,\ R_3 + 7R_2} \left(\begin{array}{ccc|c} 1 & 0 & -3 & -12 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 28 & 23 \\ 0 & 0 & 0 & 12 \end{array}\right) \cancel{\xrightarrow{R_4 + 5R_2}} \Longrightarrow \text{inconsistent}$$

$$\Longrightarrow \quad u \notin \text{span}\{v_1, v_2, v_3\}.$$

8.

$$\left(\begin{array}{ccc|c} -3 & -2 & 0 & 1 \\ 0 & 2 & -6 & 14 \\ 6 & 3 & 3 & -9 \end{array}\right) \xrightarrow{R_3 + 2R_1} \left(\begin{array}{ccc|c} -3 & -2 & 0 & 1 \\ 0 & 2 & -6 & 14 \\ 0 & -1 & 3 & -7 \end{array}\right),$$

$$\xrightarrow{R_2 + 2R_3} \left(\begin{array}{ccc|c} -3 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 3 & -7 \end{array}\right) \xrightarrow{R_1 - 2R_3} \left(\begin{array}{ccc|c} -3 & 0 & -6 & 15 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 3 & -7 \end{array}\right)$$

$$\xrightarrow[-R_2,\ 3]{-\frac{1}{3}R_1} \left(\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right) \Longrightarrow \text{consistent} \Longrightarrow$$

$$\Longrightarrow \quad p \in \text{span}\{v_1, v_2, v_3\}.$$

10. $\begin{pmatrix} -3 & -2 & 0 \\ 0 & 2 & -6 \\ 6 & 3 & 3 \end{pmatrix} \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow u \in N(A).$

12. $p = 3$, $q = 4$.

14. $\begin{pmatrix} 0 & 5 & 6 \\ 1 & -2 & 3 \\ -2 & 4 & 5 \end{pmatrix} \xrightarrow{R_3 + 2R_2} \begin{pmatrix} 0 & 5 & 6 \\ 1 & -2 & 3 \\ 0 & -10 & 11 \end{pmatrix} \xrightarrow{R_3 + 2R_1}$

$\longrightarrow \begin{pmatrix} 0 & 5 & 6 \\ 1 & -2 & 3 \\ 0 & 0 & 23 \end{pmatrix} \xrightarrow{R_{1,2}} \begin{pmatrix} 1 & -2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 23 \end{pmatrix} \Rightarrow$

each column has a column has a pivot $\Rightarrow$ column are linearly independent $\Rightarrow$ Col $A = R^3$.

But Col A = span $\left\{ \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix}, \begin{pmatrix} 6 \\ -3 \\ 5 \end{pmatrix} \right\} \Rightarrow$

$\Rightarrow$ the set of vectors is a basis for R^3 by definition.

24. Columns of A corresponding to pivot columns in echelon form of A span Col A:

$$\text{Col } A = \text{span}\left\{ \begin{pmatrix} 3 \\ -2 \\ -5 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 9 \\ 6 \end{pmatrix}, \begin{pmatrix} 3 \\ 7 \\ 3 \\ 3 \end{pmatrix} \right\}.$$

$N(A)$ - solution set of all vectors that satisfy $A\vec{x} = 0$:

$$\begin{pmatrix} 3 & -1 & 7 & 0 & 6 \\ 0 & 2 & 4 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\frac{1}{2}R_2} \xrightarrow{R_1 + R_2} \xrightarrow{\frac{1}{3}R_1}$$

$$\begin{pmatrix} 1 & 0 & 3 & 0 & 5/2 \\ 0 & 1 & 2 & 0 & 3/2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

<u>Solution:</u> $\vec{v} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = x_3 \begin{pmatrix} -3 \\ -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} -5/2 \\ -3/2 \\ 0 \\ -1 \\ 1 \end{pmatrix} \Rightarrow$

$$\Rightarrow N(A) = \text{span}\left\{ \begin{pmatrix} -3 \\ -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -5/2 \\ -3/2 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\}.$$