§ 1.8   4, 8, 10, 12, 19

MATH 230-01
ASSIGNMENT #4

4. Solve:

$$\left(\begin{array}{ccc|c} 1 & -3 & 2 & 6 \\ 0 & 1 & -4 & -7 \\ 3 & -5 & -9 & -9 \end{array}\right) \xrightarrow{R_3 - 3R_1} \left(\begin{array}{ccc|c} 1 & -3 & 2 & 6 \\ 0 & 1 & -4 & -7 \\ 0 & 4 & -15 & -27 \end{array}\right) \xrightarrow{R_3 - 4R_2}$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & -3 & 2 & 6 \\ 0 & 1 & -4 & -7 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow[R_2 + 4R_3]{R_1 - 2R_3} \left(\begin{array}{ccc|c} 1 & -3 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{R_1 + 3R_2}$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right) \Rightarrow x = \begin{pmatrix} -5 \\ -3 \\ 1 \end{pmatrix} \quad \text{unique.}$$

8. 5 rows   4 columns :

$$\underset{5\times 4}{\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ \vdots & \ddots & & \vdots \\ a_{51} & a_{52} & a_{53} & a_{54} \end{pmatrix}} \times \underset{4\times 1}{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}} = \underset{5\times 1}{\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix}}$$

10. Solve: $Ax = 0$

$$\begin{pmatrix} 1 & 3 & 9 & 2 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ -2 & 3 & 0 & 5 \end{pmatrix} \xrightarrow{R_1 - R_2} \begin{pmatrix} 0 & 3 & 6 & 6 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ -2 & 3 & 0 & 5 \end{pmatrix} \xrightarrow{R_4 + 2R_2}$$

$$\longrightarrow \begin{pmatrix} 0 & 3 & 6 & 6 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 3 & 6 & -3 \end{pmatrix} \xrightarrow{R_4 - R_1} \begin{pmatrix} 0 & 3 & 6 & 6 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -9 \end{pmatrix} \xrightarrow[\frac{1}{3} R_1]{-\frac{1}{9} R_4}$$

$$\longrightarrow \begin{pmatrix} 0 & 1 & 2 & 2 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow[\substack{R_2 + 4R_4 \\ R_3 - 3R_4}]{R_1 - 2R_4} \begin{pmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \longrightarrow$$

$$\xrightarrow{R_2 - R_1} \begin{pmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow[R_{3,4}]{R_{1,2}} \begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow x_3 \text{ free.}$$

It follows, the solution set of the above homogeneous system is the set of vectors mapped to σ:

since $\vec{x} = x_3 \cdot \begin{pmatrix} -3 \\ -2 \\ 1 \\ 0 \end{pmatrix}$ determines the solution set,

all multiples of $\vec{v} = \begin{pmatrix} -3 \\ -2 \\ 1 \\ 0 \end{pmatrix}$ are mapped by A to σ.

12. Solve:

$$\left(\begin{array}{cccc|c} 1 & 3 & 9 & 2 & -1 \\ 1 & 0 & 3 & -4 & 3 \\ 0 & 1 & 2 & 3 & -1 \\ -2 & 3 & 0 & 5 & 4 \end{array}\right) \xrightarrow{R_1 - R_2} \left(\begin{array}{cccc|c} 0 & 3 & 6 & 6 & -4 \\ 1 & 0 & 3 & -4 & 3 \\ 0 & 1 & 2 & 3 & -1 \\ -2 & 3 & 0 & 5 & 4 \end{array}\right) \xrightarrow{R_4 + 2R_2}$$

$$\longrightarrow \left(\begin{array}{cccc|c} 0 & 3 & 6 & 6 & -4 \\ 1 & 0 & 3 & -4 & 3 \\ 0 & 1 & 2 & 3 & -1 \\ 0 & 3 & 6 & -3 & 10 \end{array}\right) \xrightarrow{R_4 - R_1} \left(\begin{array}{cccc|c} 0 & 3 & 6 & 6 & -4 \\ 1 & 0 & 3 & -4 & 3 \\ 0 & 1 & 2 & 3 & -1 \\ 0 & 0 & 0 & -9 & 14 \end{array}\right) \xrightarrow{R_1 - 3R_3}$$

$$\longrightarrow \left(\begin{array}{cccc|c} 0 & 0 & 0 & -3 & -1 \\ 1 & 0 & 3 & -4 & 3 \\ 0 & 1 & 2 & 3 & -1 \\ 0 & 0 & 0 & -9 & 14 \end{array}\right) \xrightarrow{R_1 - \frac{1}{3}R_4} \left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & -17/3 \\ 1 & 0 & 3 & -4 & 3 \\ 0 & 1 & 2 & 3 & -1 \\ 0 & 0 & 0 & -9 & 14 \end{array}\right) \Rightarrow$$

$\Rightarrow$ inconsistent $\Rightarrow b \in R(A)$.

19. We have:

$$Te_1 = y_1 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

$$Te_2 = y_2 = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$$

3

Since T is linear, it is determined by a matrix:

$$Tx = Ax.$$

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Then $\begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} \Rightarrow$

$$\Rightarrow \begin{matrix} a = 2 \\ c = 5 \end{matrix}, \quad \begin{pmatrix} b \\ d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \Rightarrow \begin{matrix} b = -1 \\ d = 6 \end{matrix}.$$

It follows, $A = \begin{pmatrix} 2 & -1 \\ 5 & 6 \end{pmatrix}$.

The image of $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ under A is:

$$A\begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 5 & 6 \end{pmatrix}\begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 13 \\ 7 \end{pmatrix}.$$

Similarly,

$$\begin{pmatrix} 2 & -1 \\ 5 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2x_1 - x_2 \\ 5x_1 + 6x_2 \end{pmatrix}.$$

(24) a. <u>false:</u> see paragraph preceding example 2. (5)

b. <u>true:</u> see theorem 10

c. <u>true:</u> see table 1

d. <u>false:</u> see definition of one-to-one. Any function from $\mathbb{R}^n$ to $\mathbb{R}^m$ maps a vector onto a single (unique) vector.

e. <u>true:</u> see the solution of example 5.

27. Not one-to-one, but maps $\mathbb{R}^3$ onto $\mathbb{R}^2$.