P1

ASSIGNMENT #3

240-01

§1.5

2. $$\begin{pmatrix} 1 & -3 & 7 \\ -2 & 1 & -4 \\ 1 & 2 & 9 \end{pmatrix} \xrightarrow[R_3 - R_1]{R_2 + 2R_1} \begin{pmatrix} 1 & -3 & 7 \\ 0 & -5 & 10 \\ 0 & 5 & 2 \end{pmatrix} \xrightarrow{R_{2,3}}$$

$$\longrightarrow \begin{pmatrix} 1 & -3 & 7 \\ 0 & 5 & 2 \\ 0 & -5 & 10 \end{pmatrix} \xrightarrow{R_3 + R_2} \begin{pmatrix} 1 & -3 & 7 \\ 0 & 5 & 2 \\ 0 & 0 & 12 \end{pmatrix}$$

$\Rightarrow$ the system (homogeneous) does not have any non-trivial solutions, since there is no free variable.

4. $$\begin{pmatrix} -5 & 7 & 9 \\ 1 & -2 & 6 \end{pmatrix} \xrightarrow{R_{1,2}} \begin{pmatrix} 1 & -2 & 6 \\ -5 & 7 & 9 \end{pmatrix} \xrightarrow{R_2 + 5R_1} \begin{pmatrix} 1 & -2 & 6 \\ 0 & -3 & 39 \end{pmatrix}$$

$\Rightarrow$ x_3 is free $\Rightarrow$ the system has non trivial solution.

6. $$\begin{pmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{pmatrix} \xrightarrow[R_3 + 3R_1]{R_2 - R_1} \begin{pmatrix} 1 & 3 & -5 \\ 0 & 1 & -3 \\ 0 & 2 & -6 \end{pmatrix} \xrightarrow{R_3 - 2R_2} \begin{pmatrix} 1 & 3 & -5 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{R_1 - 3R_2} \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{pmatrix}.$$

x_3 is free. We write:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4x_3 \\ 3x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}.$$

(9) $$\begin{pmatrix} 3 & -9 & 6 \\ -1 & 3 & -2 \end{pmatrix} \xrightarrow{R_1 + 3R_2} \begin{pmatrix} 0 & 0 & 0 \\ -1 & 3 & -2 \end{pmatrix}.$$

We have: x_2, x_3 free

x_1 basic.

So $$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3x_2 - 2x_3 \\ x_2 \\ x_3 \end{pmatrix} = x_2 \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}.$$

11. $$\begin{pmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 - 3R_3} \xrightarrow{R_1 + 2R_2}$$

$$\longrightarrow \begin{pmatrix} 1 & -4 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow$$

$\Rightarrow$ x_2, x_4, x_5 free.

Write: $$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 4x_2 - 5x_5 \\ x_2 \\ x_5 \\ 4x_5 \\ x_5 \end{pmatrix} = x_2 \begin{pmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} -5 \\ 0 \\ 1 \\ 4 \\ 1 \end{pmatrix}.$$

(23) a. <u>false</u> ; see the paragraph following eq. (3). p.3
The text calls $Ax = b$ a matrix equation.

b. <u>true</u> : see box before example 3.

c. <u>false</u>: see the warning following Theorem 4

e. <u>true</u>: see parts (c) and (a) in Theorem 4

f. <u>true</u>: in Theorem 4, statement (a) is false if and only if statement (d) is false.

§ 1.7

2. <u>Solve</u>: $Ax = b$ where, $A = \cancel{\begin{pmatrix} 0 & 0 & -3 \\ 0 & 5 & 4 \\ 2 & -8 & 1 \end{pmatrix}} \longrightarrow$

$$A = \begin{pmatrix} -3 & 0 & 0 \\ 4 & 5 & 0 \\ 1 & -8 & 2 \end{pmatrix} \xrightarrow{R_2 + R_1} \begin{pmatrix} -3 & 0 & 0 \\ 1 & 5 & 0 \\ 1 & -8 & 2 \end{pmatrix} \xrightarrow{-\frac{1}{3}R_1}$$

$$\longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 1 & 5 & 0 \\ 1 & -8 & 2 \end{pmatrix} \xrightarrow[R_3 - R_1]{R_2 - R_1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & -8 & 2 \end{pmatrix} \xrightarrow{\frac{1}{5}R_2}$$

$$\longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -8 & 2 \end{pmatrix} \xrightarrow{R_3 + 8R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \xrightarrow{\frac{1}{2}R_3}$$

$$\longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \text{columns of } A$$

are linearly independent ⇒ the vectors are linearly independent.

3. $\begin{pmatrix} 1 & -3 \\ -3 & 9 \end{pmatrix} \xrightarrow{R_2 + 3R_1} \begin{pmatrix} 1 & -3 \\ 0 & 0 \end{pmatrix} \Rightarrow$ vectors are not linearly independent. In fact $\begin{pmatrix} -3 \\ 9 \end{pmatrix} = -3 \begin{pmatrix} 1 \\ -3 \end{pmatrix}$.

6. $A = \begin{pmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & -6 \end{pmatrix} \xrightarrow{R_4 - 5R_3} \begin{pmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 0 & 4 & -9 \end{pmatrix} \xrightarrow{R_4 + 4R_2}$

$\longrightarrow \begin{pmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 0 & 0 & 7 \end{pmatrix} \xrightarrow{\frac{1}{7}R_4} \begin{pmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow[R_3 - 3R_4]{R_2 - 4R_4}$

$\longrightarrow \begin{pmatrix} -4 & -3 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow$ columns of the matrix form a linearly independent set.

<u>Justification</u>: Determining if columns of A form a linearly independent set is equivalent to solving the homogeneous system $Ax = 0$ in order to find out whether the only solution to the system is the trivial solution.

8. $A = \begin{pmatrix} 1 & -3 & 3 & -2 \\ -3 & 7 & -1 & 2 \\ 0 & 1 & -4 & 3 \end{pmatrix} \xrightarrow{R_2+3R_1} \begin{pmatrix} 1 & -3 & 3 & -2 \\ 0 & -2 & 5 & -4 \\ 0 & 1 & -4 & 3 \end{pmatrix} \xrightarrow[R_1+3R_2]{R_1+3R_3}$

$\begin{pmatrix} 1 & 0 & -9 & 7 \\ 0 & 0 & -3 & 2 \\ 0 & 1 & -4 & 3 \end{pmatrix} \xrightarrow{R_{2,3}} \begin{pmatrix} 1 & 0 & -9 & 7 \\ 0 & 1 & -4 & 3 \\ 0 & 0 & -3 & 2 \end{pmatrix} \Rightarrow$

x_3 is free $\Rightarrow$ columns are linearly dependent.

(b) a. $\left(\begin{array}{cc|c} 1 & -2 & 2 \\ -5 & 10 & -9 \\ -3 & 6 & h \end{array}\right) \xrightarrow[R_3+3R_1]{R_2+5R_1} \left(\begin{array}{cc|c} 1 & -2 & 2 \\ 0 & 0 & 1 \\ -3 & 6 & h \end{array}\right) \Rightarrow$

$\Rightarrow$ the system is inconsistent for all h, i.e. for no values of h is v_3 a linear combination of v_1 and v_2.

b. $\begin{pmatrix} 1 & -2 & 2 \\ -5 & 10 & -9 \\ -3 & 6 & h \end{pmatrix} \xrightarrow[R_3+3R_1]{R_2+5R_1} \begin{pmatrix} 1 & -2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & h+6 \end{pmatrix} \Rightarrow$

$\Rightarrow x_2$ is free $\Rightarrow$ for all values of h v_1, v_2 and v_3 are linearly dependent.

(22) a. true: figure 1,

b. false: the set of $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$ is linearly dependent. See the warning after theorem 8.

c. true: see remark following example 4.

d. false: see example 3(a).