§1.3 # 6, 10, 13, 14, 17, 25

ASSIGNMENT #2

230-02

6.

$$x_1\begin{pmatrix}-2\\3\end{pmatrix} + x_2\begin{pmatrix}8\\5\end{pmatrix} + x_3\begin{pmatrix}1\\-6\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \sim$$

$$\sim \begin{pmatrix}-2 & 8 & 1\\ 3 & 5 & -6\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \sim$$

$$\sim \begin{aligned} -2x_1 + 8x_2 + x_3 &= 0\\ 3x_1 + 5x_2 + 6x_3 &= 0\end{aligned}$$

10.

$$\begin{aligned} 4x_1 + x_2 + 3x_3 &= 9\\ x_1 - 7x_2 - 2x_3 &= 2\\ 8x_1 + 6x_2 - 5x_3 &= 15\end{aligned} \sim$$

$$\sim x_1\begin{pmatrix}4\\1\\8\end{pmatrix} + x_2\begin{pmatrix}1\\-7\\6\end{pmatrix} + x_3\begin{pmatrix}3\\-2\\-5\end{pmatrix} = \begin{pmatrix}9\\2\\15\end{pmatrix}.$$

13.

$$A = \begin{pmatrix}1 & -4 & 0\\ 0 & 3 & 5\\ -2 & 8 & -4\end{pmatrix} \qquad b = \begin{pmatrix}3\\-7\\-3\end{pmatrix}$$

If the matrix equation $Ax = b$ has a unique solution $x = (x_1, x_2, x_3)$ then b can be written as a linear combination of columns of A.

So we consider and solve the augmented matrix:

$$\left(\begin{array}{ccc|c} 1 & -4 & 0 & 3\\ 0 & 3 & 5 & -7\\ -2 & 8 & -4 & -3\end{array}\right) \xrightarrow{R_3 + 2R_1}$$

13.

$$\xrightarrow{R_3+2R_1} \left(\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{array}\right) \Rightarrow$$ the system is inconsistent $\Rightarrow$

$\Rightarrow$ columns of A do not form a linear combination =

= b.

(14) If $Ax = b$ has a unique solution, b is a linear combination of columns of A:

$$\left(\begin{array}{ccc|c} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{array}\right) \xrightarrow{R_3-R_1} \left(\begin{array}{ccc|c} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 0 & 0 & 11 & -2 \end{array}\right)$$

$\Rightarrow$ the system has a unique solution $\Rightarrow$ b is a linear combination of columns of A.

(P.)

$$\left(\begin{array}{cc|c} 1 & -2 & 4 \\ 4 & -3 & 1 \\ -2 & 7 & h \end{array}\right) \xrightarrow[R_3+2R_1]{R_2-4R_1} \left(\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 5 & -15 \\ 0 & 3 & h+8 \end{array}\right) \xrightarrow{R_2 \cdot 1/5}$$

$$\rightarrow \left(\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 3 & h+8 \end{array}\right) \rightarrow$$ $x_2 = -3$ so that $3x_2 = h+8 = -9 \rightarrow$

$\rightarrow h = -17$.

In other words, when the 3^{rd} row = 2^{nd} row.

25. a. No.

There are 3 vectors in $\{a_1, a_2, a_3\}$:

$\{a_1, a_2, a_3\}$ is a set.

b.

$$\left(\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ -2 & 6 & 3 & -4 \end{array}\right) \xrightarrow{R_3+2R_1} \left(\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 6 & -5 & 4 \end{array}\right)$$

$$\xrightarrow{R_3-2R_2} \left(\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & -1 & 2 \end{array}\right) \xrightarrow[R_1-4R_3]{R_2-2R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 3 & 0 & -3 \\ 0 & 0 & -1 & 2 \end{array}\right) \Rightarrow$$

$\Rightarrow \quad b \in \operatorname{span}\{a_1, a_2, a_3\} = W$

There are infinitely many vectors in $\operatorname{span}\{a_1, a_2, a_3\}$

c. $a_1 = 1 \cdot a_1 + 0 \cdot a_2 + 0 \cdot a_3 \Rightarrow a_1 \in W$.

§ 1.4.

2. Undefined: # of columns (1) in the 3×1 matrix does not match # of entries (2) in the vector.

3. $$\begin{pmatrix} 6 & 5 \\ -4 & -3 \\ 7 & 6 \end{pmatrix}\begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ -4 \end{pmatrix}.$$

10. $$\begin{aligned} 8x_1 - x_2 &= 4 \\ 5x_1 + 4x_2 &= 1 \\ x_1 + 3x_2 &= 2 \end{aligned} \quad \sim \quad x_1\begin{pmatrix} 8 \\ 5 \\ 1 \end{pmatrix} + x_2\begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 8 & -1 \\ 5 & 4 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}.$$

11.

$$\left(\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ -2 & -4 & -3 & 9 \end{array}\right) \xrightarrow{R_3+2R_1} \left(\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 5 & 5 \end{array}\right) \xrightarrow{1/5\, R_3}$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{R_2-5R_3} \left(\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow[R_1-4R_3]{R_1-2R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

Solution in vector form: $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}$.

18. Only 3 rows of B contain pivot positions $\Rightarrow$ $Bx = y$ does not have a solution for each $y \in \mathbb{R}^4$ by <u>Theorem 4.</u>

21. No: the matrix $[v_1\ v_2\ v_3]$ does not have a pivot in each row $\Rightarrow$ span$\{v_1, v_2, v_3\} \neq \mathbb{R}^4$.

18.

$$\begin{pmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{pmatrix} \xrightarrow[R_4+2R_1]{R_3-R_1} \begin{pmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & -1 & -1 & 5 \\ 0 & -2 & -2 & 3 \end{pmatrix} \xrightarrow{R_3+R_2}$$

$$\longrightarrow \begin{array}{c} \begin{array}{cccc} \underline{b_1} & \underline{b_2} & \underline{b_3} & \underline{b_4} \end{array} \\ \begin{pmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ \boxed{0 \quad 0 \quad 0 \quad 0} \\ 0 & -2 & -2 & 3 \end{pmatrix} \end{array}$$

3rd row contains no pivot $\Rightarrow$

$\Rightarrow$ span$\{b_1, b_2, b_3, b_4\} \neq \mathbb{R}^4$.

<u>Note:</u> B is <u>not</u> an augmented matrix!