

$$1. a. \begin{pmatrix} 1 & 5 & | & 7 \\ -2 & -7 & | & -5 \end{pmatrix} \xrightarrow{R_2 + 2R_1} \begin{pmatrix} 1 & 5 & | & 7 \\ 0 & 3 & | & 9 \end{pmatrix} \xrightarrow{\frac{1}{3}R_2} \begin{pmatrix} 1 & 5 & | & 7 \\ 0 & 1 & | & 3 \end{pmatrix} \quad (1)$$

$$\xrightarrow{R_1 - 5R_2} \begin{pmatrix} 1 & 0 & | & -8 \\ 0 & 1 & | & 3 \end{pmatrix} \Rightarrow \begin{matrix} x_1 = -8 \\ x_2 = 3 \end{matrix}$$

ASSIGNMENT #1

230-01

3. In order to find the pt. of intersection solve the following system, represented by the augmented matrix:

$$\begin{pmatrix} 1 & 5 & | & 7 \\ 1 & -2 & | & -2 \end{pmatrix} \xrightarrow{R_2 - R_1} \begin{pmatrix} 1 & 5 & | & 7 \\ 0 & -7 & | & -9 \end{pmatrix} \xrightarrow{-\frac{1}{7}R_2} \\ \rightarrow \begin{pmatrix} 1 & 5 & | & 7 \\ 0 & 1 & | & 9/7 \end{pmatrix} \xrightarrow{R_1 - 5R_2} \begin{pmatrix} 1 & 0 & | & 4/7 \\ 0 & 1 & | & 9/7 \end{pmatrix} \Rightarrow$$

\Rightarrow the pt. of intersection of the two lines is $(x_1, x_2) = (4/7, 9/7)$.

6. Replace row 4 by its sum with -3 times row 3. After that, scale row 4 by $-1/5$.

II. Inconsistent:

$$\begin{pmatrix} 0 & 1 & 4 & | & -5 \\ 1 & 3 & 5 & | & -2 \\ 3 & 7 & 7 & | & 6 \end{pmatrix} \xrightarrow{R_3 - 3R_2} \begin{pmatrix} 0 & 1 & 4 & | & -5 \\ 1 & 3 & 5 & | & -2 \\ 0 & -2 & -8 & | & 12 \end{pmatrix} \xrightarrow{R_3 + 2R_1} \\ \rightarrow \begin{pmatrix} 0 & 1 & 4 & | & -5 \\ 1 & 3 & 5 & | & -2 \\ 0 & 0 & 0 & | & 2 \end{pmatrix} \Rightarrow 0 = 2 - \text{a contradiction} \Rightarrow \\ \text{the system is inconsistent.}$$

15.
$$\left(\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{array} \right) \xrightarrow{R_4 - 3R_1} \left(\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 0 & 0 & -9 & 7 & -11 \end{array} \right) \quad (2)$$

$$\xrightarrow{R_3 + 2R_2} \left(\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & -9 & 7 & -11 \end{array} \right) \xrightarrow{R_4 + 3R_3} \left(\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & 0 & -5 & 10 \end{array} \right) \Rightarrow$$

→ the system is consistent.

16.
$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{array} \right) \xrightarrow{\frac{1}{2}R_2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{array} \right)$$

$$\xrightarrow{R_4 + 2R_1} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -3 & -1 \end{array} \right) \xrightarrow{R_4 - 2R_3} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 0 & -9 & -5 \end{array} \right) \xrightarrow{\frac{1}{3}R_4}$$

$$\begin{aligned} &\rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 1 & 0 & -3 & -1 \end{array} \right) \xrightarrow{R_2 - R_4} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 1 & 0 & -3 & -1 \end{array} \right) \quad (3) \\ &\xrightarrow{R_3 - R_2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 & -1 \end{array} \right) \xrightarrow{R_{3,4}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 1 & 0 & -3 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{R_{2,3}} \\ &\rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \Rightarrow \text{the system is consistent.} \end{aligned}$$

§ 1.2

(2.) Reduced Echelon form:

a

Echelon form:

b, d

Not Echelon:

c

(3.) $\left(\begin{array}{ccc|c} \boxed{1} & 0 & -1 & -2 \\ 0 & \boxed{1} & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right)$ pivot columns are: 1 and 2.

(8.) $\begin{cases} x_1 = -9 \\ x_2 = 4 \\ x_3 \text{ is free} \end{cases}$

$$\begin{aligned} &\left(\begin{array}{ccc|c} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{array} \right) \xrightarrow{R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 4 & 0 & 7 \\ 0 & -1 & 0 & -4 \end{array} \right) \xrightarrow{-R_2} \left(\begin{array}{ccc|c} 1 & 4 & 0 & 7 \\ 0 & 1 & 0 & 4 \end{array} \right) \rightarrow \\ &\xrightarrow{R_1 - 4R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & 4 \end{array} \right) \end{aligned}$$

$$(14.) \left(\begin{array}{ccccc|c} 1 & 2 & -5 & -6 & 0 & -5 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{R_1 - 2R_2} \left(\begin{array}{ccccc|c} 1 & 0 & 7 & 0 & 0 & -9 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \quad (4)$$

$\Rightarrow x_1, x_2, x_5$ are basic variables
 x_3, x_4 are free variables

The general solution is:

$$\begin{cases} x_1 = -9 - 7x_3 \\ x_2 = 2 + 6x_3 + 3x_4 \\ x_3 \text{ free} \\ x_4 \text{ free} \\ x_5 = 0 \end{cases}$$

(15.) a. Consistent with a unique solution

b. Inconsistent, $0 \neq \blacksquare$.

(16.) a. Consistent with a unique solution

b. Consistent with infinitely many solutions:
 column 2 is non-basic.

(18.) $h \neq -15$:

$$\left(\begin{array}{cc|c} 1 & -3 & -2 \\ 5 & h & -7 \end{array} \right) \xrightarrow{R_2 - 5R_1} \left(\begin{array}{cc|c} 1 & -3 & -2 \\ 0 & h+15 & 3 \end{array} \right) \Rightarrow$$

\Rightarrow inconsistent if $0 = 3$, i.e. if $h+15 = 0 \Rightarrow$

\Rightarrow consistent if $h \neq -15$.