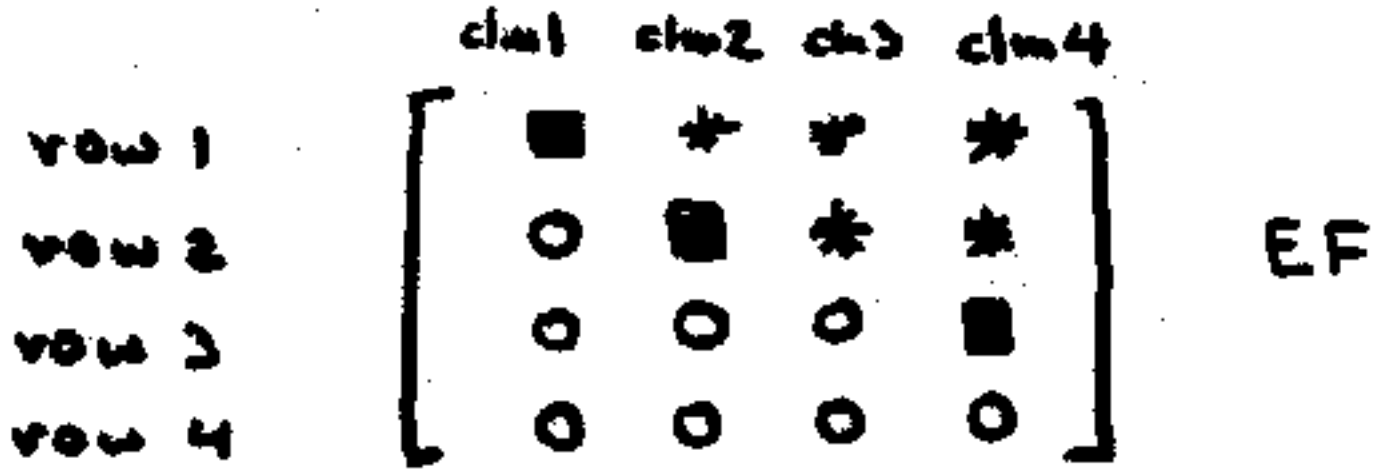
LEADING ENTRY = left-most nonzero entry in a nonzero row

ECHELON FORM (EF)
of an $m \times n$ augmented matrix

1. all nonzero rows are above any zero rows.

2. each leading entry of a row is in a column to the right of the leading entry of the row above it.

3. all entries in a column below a leading entry are <u>zero</u>



EF

■ = leading entry

* = anything

REDUCED ECHELON FORM (REF)

It is an EF and in addition

4. the leading entry in each row equals <u>one</u>

5. each leading entry is the <u>only</u> nonzero entry in its column

$$A \sim \begin{bmatrix} \textcircled{1} & 0 & * & * & 0 \\ 0 & \textcircled{1} & * & * & 0 \\ 0 & 0 & 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

REF of A

↑ ↑ ↑ pivot columns

PIVOT POSITION in a matrix A is a location in A that corresponds to a leading entry in the EF (or REF) of A.
The corresponding column is a PIVOT COLUMN

STRATEGY TO CREATE EF / REF
(GAUSSIAN ELIMINATION)

STEP 1 Begin with left-most nonzero column (this is a <u>pivot column</u>)

STEP 2 Select a nonzero entry in the pivot column as the <u>pivot.</u>
(if necessary interchange rows to move pivot in pivot position)

$$\begin{bmatrix} \boxed{0} & 2 & 3 & 4 \\ \boxed{1} & 2 & 0 & 1 \\ \boxed{6} & 0 & 0 & 6 \end{bmatrix} \xrightarrow{P_{12}} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 2 & 3 & 4 \\ 6 & 0 & 0 & 6 \end{bmatrix}$$

STEP 3 Use row replacement (add multiples of the row with pivot to others to create zeros below pivot)

$$\xrightarrow{R_3 - 6R_1} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & \boxed{2 & 3 & 4} \\ 0 & \boxed{-12 & 0 & 0} \end{bmatrix}$$

STEP 4 Repeat process to submatrix $\boxed{\quad}$

$$\xrightarrow{R_3 + 6R_2} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 18 & 24 \end{bmatrix}$$ EF!

REF

STEP 5 (to create REF)

begin with right-most pivot, make it one by scaling and use it to create zeros above it

$\xrightarrow{1/18 R_3}$ $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 1 & 4/3 \end{bmatrix}$

$\xrightarrow{R_2 - 3R_3}$ $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 4/3 \end{bmatrix}$

$\xrightarrow{1/2 R_2}$ $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4/3 \end{bmatrix}$

$\xrightarrow{R_1 - 2R_2}$ $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4/3 \end{bmatrix}$ REF!

(If we were solving a system:

$x_1 = 1$, $x_2 = 0$, $x_3 = 4/3$)