Exam I

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Name: ________________  Section: ______

ID Number: ____________________________________________

Instructions: Work must be shown to receive credit. No calculators, books or notes. Please circle your answers.

1. (a) (10 pts) Suppose you know only that \( \lim_{(x,y) \to (3,7)} g(x,y) = 12 \). What can you say about \( g(3,7) \)?

\[ \text{nothing} \]

(b) (10 pts) What if you know that \( \lim_{(x,y) \to (4,-2)} h(x,y) = 9 \) and \( h \) is continuous at \((4, -2)\). What can you say about \( h(4, -2) \)?

\[ h(4, -2) = 9 \]
2(a). (10 points) Let $f(x, y)$ be a function of two variables. Give the limit definition of the following partial derivative.

$$f_x(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0 + h, y_0) - f(x_0, y_0)}{h}$$

Evaluate the following for the function $f(x, y) = 3x^2 + 2xy$

(b) (5 points) $\frac{\partial f}{\partial x} (x, y) = 6x + 2y$

(c) (5 points) $\frac{\partial f}{\partial x} (1,5) = 6 \cdot 1 + 2 \cdot 5 = 16$
3. (20 points) Find an equation of the plane tangent to the surface of
\[ f(x, y) = 2x^2 - y^2 \]
at the point \((x, y) = (1, 1)\)

\[ z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) \]

\[ z_0 = f(1, 1) = 1 \]
\[ f_x = 4x \]
\[ f_x(1, 1) = 4 \]
\[ f_y = -2y \]
\[ f_y(1, 1) = -2 \]

\[ z - 1 = 4(x - 1) - 2(y - 1) \]
4. Let \( f(x, y) = e^x \cos(y) \) where \( x = st \) and \( y = s/t \).

(a) (15 pts) Use the chain rule to find \( \frac{\partial f}{\partial t} \) as a function of \( s \) and \( t \).

\[
\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial t}
\]

\[
= (e^x \cos(y))s - (e^x \sin(y))\left(\frac{s}{t^2}\right)
\]

\[
\frac{\partial f}{\partial t} = e^{st} \cos\left(\frac{s}{t}\right)s - e^{st} \sin\left(\frac{s}{t}\right)\left(\frac{s}{t^2}\right)
\]

(b) (5 pts) Evaluate \( \frac{\partial f}{\partial t} \) when \( s = \pi \) and \( t = 1 \).

\[
\left. \frac{\partial f}{\partial t} \right|_{s=\pi, t=1} = e^{\pi \cos(\pi)} \pi - e^{\pi \sin(\pi)}(-\pi)
\]

\[
= -\pi e^\pi - 0
\]
5. Let \( f(x, y) = x^2 + xy^2 \) and let \( P \) be a point with coordinates \((2, 1)\).

(a) (10 pts) Find \( \nabla f(2, 1) \)

\[
\nabla f(x, y) = \langle 2x + y^2, 2xy \rangle \\
\n\nabla f(2, 1) = \langle 4 + 1, 2(2)(1) \rangle = \langle 5, 4 \rangle
\]

(b) (5 pts) Find the maximum rate of change of the function of \( f \) at the point \( P \). The answer should be a real number.

\[
\text{Max ROC} = |\nabla f(2, 1)| \\
= \sqrt{25 + 16} = \sqrt{41}
\]

(c) (5 pts) Find the rate of change of \( f \) at the point \( P \) in the direction \( \hat{u} = (1, -3)/\sqrt{10} \).

\[
\nabla \hat{u} f(2, 1) = \nabla f(2, 1) \cdot \hat{u} \\
= \langle 5, 4 \rangle \cdot \frac{1}{\sqrt{10}} \langle 1, -3 \rangle \\
= \frac{1}{\sqrt{10}} \cdot (5 - 12) = \frac{-7}{\sqrt{10}}
\]