Differential Equations

Ex: \[ y' + y = t \]
or \[ y = f(t), \text{ and we want to find an } f(t) \]
such that

\[ f'(t) + f(t) = t \]
for all \( t \) values in some interval.

Can be interpreted as:

\[ y' + y = t \]
\[ y' = t - y \]
The slope of a solution curve is given by $t - y$.

The typical solutions to $y' + y = t$ are shown.

A table is also provided:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$y$</th>
<th>$y' = t - y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>-1</td>
</tr>
</tbody>
</table>
A diff eqn. with a initial condition is called an Initial Value Problem (IVP):

\[ y' + y = t, \quad y(0) = 2 \quad \Rightarrow \quad y(t) \]

Let's actually solve some DE's / IVP's:
1.3 (5) \[ y' = \frac{t}{1+t^2} \]. Find general sol.

Integrate both sides:

\[
\int y' \, dt = \int \frac{t}{1+t^2} \, dt
\]

\[ y(t) = \frac{1}{2} \ln (1+t^2) + C \]
2.1 Differential Eqs: First-order.

Order of DE is highest deriv. order in the eqn:

\[ y'y + t = e^t \quad \text{1st order} \]

\[ yy'' - y' + e^t = t^2 \quad \text{2nd order} \]

\[ 5y^{(4)} + 2y'' - y = 5 \quad \text{4th order}. \]
Normal form for DE means it's in the form of

\[ y^{(\text{highest deriv})} = \text{lower y-terms, t-stuff}. \]

**Ex:** normal form for \( yy' + t = e^t \) is

\[ y' = \frac{e^t - t}{y} \]

Note: The generic version of a 1st order IVUP is:
\[ y' = f(t, y), \quad y(t_0) = y_0 \]

Partial Diff. eqns are ones involving partial derivs, like:

\[ \frac{dy}{dx} + \frac{dy}{dw} = w^2 + x \]

(here, \( y \) is func of \( w + x \))

In this class, we deal with Ordinary Diff Eqns:

\[ \frac{dy}{dt} = y^2 + t \]
Interval of existence