1. [20] Consider an encryption scheme which encrypts each letter of a message by converting the letter to a number (A-Z become 0-25, respectively), then applying the function \( f(p) = 11p + 10 \mod 26 \), and then converting the resulting number back to a letter. For example, the letter C would be encrypted as follows:

   C is converted to 2, then \( f(2) = 11(2) + 10 \mod 26 = 32 \mod 26 = 6 \), which then converts to G.

a) Find \( 11 \cdot 19 \mod 26 \). (this is supposed to help you with parts (b) and (c))

b) Find out which letter would encrypt to letter O by solving \( 11p + 10 \equiv 14 \mod 26 \).

c) Find a decryption function by solving \( 11p + 10 \equiv n \mod 26 \) for \( p \).
2. [10] Let $P(n)$ be the statement that a postage of $n$ cents can be formed using just 3-cent stamps and 7-cent stamps. Use induction (strong or weak) to prove that $P(n)$ is true for $n \geq 12$.

3. [10] Show that if $A$ and $B$ are any two countably infinite sets, then $A \times B$ is countable.
4. [10] Finish the following definitions and statements. Assume $a, b, c$ are positive integers.

a) A positive integer $p$ is prime iff ________________

b) If $a \equiv b \pmod{c}$, then ________________

c) $a$ and $b$ are relatively prime iff ________________

d) If $a|b$ and $b|c$, then ________________

e) If ___________ and ___________, then $a|(sb + tc)$ for all $s, t \in \mathbb{Z}$.

5. [10] Find the value of the double sum $\sum_{i=1}^{2} \sum_{j=1}^{4} \frac{i}{j}$. (give answer as a single fraction or mixed number)

6. [10] Use the Euclidean algorithm to find $\gcd(201, 111)$. (show all steps)