Math 567: Integer and Combinatorial Optimization (Spring 2015)

Time       Tue-Thu 12:00–1:15 pm
Location   Murrow 52 (via AMS) — now called Jackson 52
Instructor Bala Krishnamoorthy
Office     Neill 325 (in Pullman), Skype and Google ID: wsucomptopo
Office Hours Tue 2–3 pm, Wed 2–3 pm, or by appointment
Email      bkrishna@math.wsu.edu
Web page   http://www.wsu.edu/~kbala/Math567.html
Text       Class notes and handouts
References Dimitris Bertsimas and Robert Weismantel: Optimization over Integers
Laurence A. Wolsey: Integer Programming

Description of the Course
Solving optimization problems with variables restricted to take integer values, as opposed to real values, is called integer optimization. The subject, also commonly called integer programming (IP), uses concepts from various areas of mathematics and computer science including linear algebra, combinatorics, geometry of numbers, algebraic geometry, as well as algorithms and data structures. IP techniques have been used to model and solve problems in electrical power systems, airline crew scheduling, economic lot-sizing, transportation and logistics, treatment of tumors using radiation, computational biology, and many other areas.

This graduate level course aims to provide a detailed treatment of the theory, solution methods, and applications of integer and combinatorial optimization. Topics covered could include IP formulations, binary expressions and conjunctive normal form (CNF), enumerative methods (branch-and-bound), theory of cutting planes, lattice-based approaches including basis reduction, algebraic geometry methods, polyhedral geometry, and computational complexity. We will also emphasize the use of state-of-the-art software packages to model and solve real-life problems. The packages AMPL and CPLEX will be introduced through several homework problems and a project.

As a prerequisite, students should have taken an undergraduate level course in Linear Optimization (MATH 364, MATH 464, or equivalent), or obtain the permission of the instructor. I can make exceptions on a case-by-case basis for this requirement. Familiarity with programming language(s), or packages such as MATLAB will be helpful, but not required.

Organization and Grading
The course will have around eight homework assignments. These assignments will include theoretical problems as well as ones that involve the use of software packages. Each assignment will be handed out at least a week before the day on which it will be due. Apart from the homework assignments, there will be two course projects (involving AMPL/CPLEX and/or MATLAB). The total score for the course will be calculated using the following weights: homework - 60%, projects - 40%.

Academic Integrity: Discussion of homework problems with others is allowed, and is also encouraged. But each person should hand in his or her own written solutions. Plagiarism or cheating will not be tolerated. Such behavior will result in a zero grade for a graded item and possibly a failing grade for the entire course.

Students with Disabilities: Reasonable accommodations are available for students with a documented disability through the Access center in Pullman. All accommodations MUST be approved through the Access Center (Pullman, Washington Building, Room 217). Please stop by or call 509-335-3417 for the Access Advisor.

WSU Safety Measures: Washington State University is committed to maintaining a safe environment for its faculty, staff, and students. Please visit safetyplan.wsu.edu and oem.wsu.edu/emergencies to access
the Campus Safety Plan and emergency information You should also become familiar with the WSU Alert Site (alert.wsu.edu) where information about emergencies and other issues affecting WSU will be found.

Software

The modeling language AMPL (www.ampl.com) along with the solver CPLEX will be used in the course. Students will model several examples as well as real-life integer optimization problems using this software. Further analysis could be done using MATLAB. The course project will involve creating a complete model for one or two real-life instances of integer optimization problems, applying heuristics, and interpreting the solutions obtained.

Topics covered

*The following is a rough plan. As the course progresses, I may include new topics and/or delete some of the ones listed here.*

1. Integer programming basics and formulations - 5 lectures
   - IP formulations, modeling with binary variables, facility location, traveling salesman and other network problems, disjunctive constraints
   - binary expressions - conjunctive normal form (CNF)
   - strength of formulations, aggregated and disaggregated formulations

2. Solvers and applications - 2 lectures
   - Introduction to commercial software packages (AMPL/CPLEX)
   - the p-median and p-center problems, fixed charge network flow, local area network (LAN) planning

3. Enumerative methods - branch-and-bound - 2 lectures

4. Theory of valid inequalities - 4 lectures
   - Chvátal-Gomory (CG) cuts, mixed-integer rounding, disjunctive cuts
   - knapsack cover inequalities, Lovász-Schrijver procedure

5. Lattices and applications - 5 lectures
   - Hermite normal form (HNF) and Diophantine equations
   - basis reduction (BR), Lenstra’s algorithm, BR-based reformulation techniques
   - shortest vector problem (SVP), closest vector problem (CVP)

6. Algebraic geometry techniques - 5 lectures
   - ideals, varieties, Gröbner basis
   - Gröbner basis algorithms for binary and integer optimization
   - generating functions for integer points in polyhedra

7. Computational complexity - 2 lectures
   - NP-completeness, complexity, and polyhedra

8. Integral polyhedra, matching, other topics - 5 lectures
   - totally unimodular matrices
   - network matrices, balanced and totally balanced matrices, maximum-cardinality and maximum-weight matching