

Math 567: Integer and Combinatorial Optimization (Spring 2009)

Time	Tue-Thu 12:00–1:15 pm
Location	Webster B12
Instructor	Bala Krishnamoorthy
Office	Neill 325
Office Hours	Tue 2:00–4:00 pm, Wed 11:30 am–1:30 pm
Email	kbala@wsu.edu
Web page	http://www.wsu.edu/~kbala/Math567.html
Text	Class notes and handouts
References	Laurence A. Wolsey: Integer Programming John Wiley and Sons, ISBN: 0-471-28366-5.

Dimitris Bertsimas and Robert Weismantel: Optimization over Integers
Dynamic Ideas, ISBN: 0-9759146-2-6.

George L. Nemhauser and L.A. Wolsey: Integer and Combinatorial Optimization
John Wiley and Sons, ISBN: 0471-35943-2.

Description of the Course

Solving optimization problems with variables restricted to take integer values, as opposed to real values, is called integer optimization. The subject, also commonly called integer programming (IP), has seen major developments since early 1960s. It uses concepts from various areas of mathematics and computer science including linear algebra, algebraic geometry, combinatorics, geometry of numbers, algorithms and data structures. IP techniques have been used to model and solve problems in electrical power systems, airline crew scheduling, economic lot-sizing, transportation and logistics, treatment of tumors using radiation, and computational biology, to name a few.

This graduate level course aims to provide a detailed treatment of the theory, solution methods, and applications of integer and combinatorial optimization. Topics covered will include IP formulations, binary expressions and conjunctive normal form (CNF), enumerative methods (branch-and-bound), theory of cutting planes, lattice-based approaches including basis reduction, algebraic geometry methods, polyhedral geometry, and computational complexity. We will also emphasize the use of state-of-the-art software packages to model and solve real-life problems. The packages AMPL and CPLEX will be introduced through several homework problems and a project.

As a prerequisite, students should have taken a course in Linear Optimization (MATH 464 or equivalent), or obtained the permission of the instructor. Familiarity with programming language(s), or packages such as MATLAB will be helpful, but *not* required.

Organization and Grading

The course will have around ten homework assignments. These assignments will include theoretical problems as well as ones that involve the use of software packages. Each assignment will be handed out at least a week before the day on which it will be due. There will be one exam, which will be the final exam. This exam will be a take-home, open-book exam. Apart from the homework assignments and the final exam, there will be a course project (involving AMPL/CPLEX and MATLAB). The total score for the course will be calculated using the following weights: homework - 50%, project - 25%, final exam - 25%.

Discussion of homework problems with others is allowed, but each person should hand in his or her own written solutions. Plagiarism or cheating will not be tolerated. Such behavior will result in a zero grade for a graded item and possibly a failing grade for the entire course.

Please notify the instructor during the first week of class of any accommodations needed for a documented disability during the course. All accommodations must be approved through the Disability Resource Center (DRC) in Administration Annex room 206, 335-1566, drc@mail.wsu.edu.

Software

The modeling language AMPL (www.ampl.com) along with the solver CPLEX will be used in the course. Students will model several examples as well as real-life integer optimization problems using this software. Further analysis could be done using MATLAB. The course project will involve creating a complete model for one or two real-life instances of integer optimization problems, applying heuristics, and interpreting the solutions obtained.

Topics covered

The following is a rough plan. As the course progresses, I may include new topics and/or delete some of the ones listed here.

1. [Integer programming basics and formulations](#) - 5 lectures
 - IP formulations, modeling with binary variables, facility location, traveling salesman and other network problems, disjunctive constraints
 - binary expressions - conjunctive normal form (CNF)
 - strength of formulations, aggregated and disaggregated formulations
2. [Solvers and applications](#) - 2 lectures
 - Introduction to commercial software packages (AMPL/CPLEX)
 - the p-median and p-center problems, fixed charge network flow, local area network (LAN) planning
3. [Enumerative methods](#) - branch-and-bound - 2 lectures
4. [Theory of valid inequalities](#) - 4 lectures
 - Chvátal-Gomory (CG) cuts, mixed-integer rounding, disjunctive cuts
 - knapsack cover inequalities, Lovász-Schrijver procedure
5. [Lattices and applications](#) - 5 lectures
 - Hermite normal form (HNF) and Diophantine equations
 - basis reduction (BR), Lenstra's algorithm, BR-based reformulation techniques
 - shortest vector problem (SVP), closest vector problem (CVP)
6. [Algebraic geometry techniques](#) - 5 lectures
 - ideals, varieties, Gröbner basis
 - Gröbner basis algorithms for binary and integer optimization
 - generating functions for integer points in polyhedra
7. [Computational complexity](#) - 2 lectures
 - NP-completeness, complexity, and polyhedra
8. [Integral polyhedra, matching, other topics](#) - 4 lectures
 - totally unimodular matrices
 - network matrices, balanced and totally balanced matrices, maximum-cardinality and maximum-weight matching