# Integer Optimization (Spring 2009) — Homework 4

- For problems marked with [**A**], you need to use AMPL and CPLEX to solve the problem. You must send me (by email to bkrishna@math.wsu.edu) your model and data files. The name of your files must indicate who you are – so, if you are Eric Cartman, you could name the model file for problem 1 as Prob1_Eric_Cartman.mod and so on.
- The total points (given in parentheses) add up to 130. You will be graded for 120 points.
- **This homework is due in class on Tuesday, February 17**.

1. (30) [**A**] Solve the instance of *capacitated* lot sizing problem over $T = 12$ time periods with demands $\boldsymbol{d} = [7,5,4,8,10,13,8,5,10,12,17,7]$, fixed (start-up) costs $\boldsymbol{f} = [12,14,30,13,15,45,22,15,17,14,30,19]$, unit production costs $\boldsymbol{c} = [5,3,3,4,6,3,2,4,5,\ 3,3,4]$, unit holding costs $\boldsymbol{h} = [1,2,2,1,2,1,3,2,2,2,3,1]$, and a maximum production capacity of 10 units in each period. Your model file should be fully independent of the data, i.e., it should work without having to make any changes when the data changes in *any* fashion (including a change in the number of time periods). Also, use $M_t = 10000$ for each $t$, instead of the smallest values that work, and comment on any difference in solution times or branch-and-bound nodes observed.

2. (30) [**A**] We want to assign digits (or numbers) to the letters $W, D, O, T, G, L, E, C, M$ from out of 0–9 so that the following subtraction is true:

$$\begin{array}{cccccc} W & W & W & D & O & T\ - \\ G & O & O & G & L & E\ = \\ \hline D & O & T & C & O & M \\ \hline \end{array}$$

   There are nine letters, and each letter must be assigned a different number. Formulate this problem as an integer program and solve it using AMPL.

3. (30) [**A**] Write an AMPL model to solve the queens placement problem discussed in Problem 5 of Homework 1. Find the maximum number of queens that can be placed without mutual attacks for $N = 8, 16, 20, 32$. Note that for larger values of $N$ (18 or more), the student version of AMPL will not be sufficient, as it has a limit of 300 variables and 300 constraints. You should use the full version of AMPL/CPLEX available in the Math Department machine `thetahat`.

4. (30) Let $S$ be the set of tours in the directed graph $G = (V, E)$ with $|V| = n$ (representations of this set were discussed in class). The first set of constraints (named as equations (1) in class) are

$$\begin{array}{rcl} \sum_j x_{ij} & = & 1 \ \ \forall\, i, \\ \sum_j x_{ji} & = & 1 \ \ \forall\, i, \\ 0 & \le x_{ij} \le & 1, \quad x_{ij} \in \mathbb{Z}. \end{array} \tag{4.1}$$

   Now consider the following system:

$$\begin{array}{rcl} \sum_j y_{ij} - \sum_j y_{ji} & = & -1 \ \ \forall\, i \neq 1, \\ \sum_j z_{ij} - \sum_j z_{ji} & = & 1 \ \ \forall\, i \neq 1, \\ y \ge 0, \quad z \ge 0. & & \end{array} \tag{4.2}$$

   Prove that

$$S = \{ \boldsymbol{x} \mid b \text{ satisfies (4.1)}, \exists \boldsymbol{y} \in \mathbb{R}^{|E|}, \boldsymbol{z} \in \mathbb{R}^{|E|} \text{ such that } (\boldsymbol{y}, \boldsymbol{z}) \text{ satisfies (4.2) and } \boldsymbol{x} = \frac{1}{n}(\boldsymbol{y}+\boldsymbol{z}) \,\}.$$