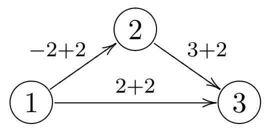
Network Optimization (Fall 2008) – Brief Solutions to Homework 8

1. Apply the label correcting algorithm starting with the obtained "optimal" distance labels and actual costs c'_{ij}. The re-optimization will be complete after $O(L)$ passes. The total changes in all $d(i)$ values (or, in $\sum_{i \in N} d(i)$) will be at most L, and in each pass except the last one, at least one $d(i)$ value is decreased by at least 1.

2. The claim is wrong. In the following network, if we add $|c_{\min}| = 2$ units to each arc, the shortest path to node 3 from node 1 changes from 1-2-3 to 1-3.



3. When c_{pq} is decreased to c'_{pq}, any i-j shortest path that originally used (p, q) will continue to use it. Further, some i-j shortest paths that did not use (p, q) originally may now include it. In both cases, d_{ij} can be updated in $O(1)$ time (as described in Problem 5.40, AMO page 163). On the other hand, if $c'_{pq} > c_{pq}$, it may affect the d_{ij} values for an i-j shortest path that uses (p, q), but it does not seem that this change can be estimated in $O(1)$ time. So, the re-optimization cannot be achieved in $O(n^2)$ time.

4. Set the capacity of each arc $(i, j) \in A$ as $u_{ij} = \lfloor \lambda / c_{ij} \rfloor$. Then solve a feasible flow problem on this modified network, which can be modeled as a max-flow problem.

5. Replace each undirected arc (i, j) in the graph with two directed arcs (i, j) and (j, i) both having unit capacity. Select any node as the source node s. Find the min-cut with the sink node $t = i$ for each $i \in N \setminus \{s\}$. Thus you'll have to solve $n - 1$ max-flow problems. The minimum number of arcs in any partition is given by the value of the smallest min-cut among all $n - 1$ min-cuts.

6. Let there be k source nodes $\{s_1, \ldots, s_k\}$, and k sink nodes $\{t_1, \ldots, t_k\}$. Add another source node s, a sink node t, arcs (s, s_i) , and arcs (t_i, t). Set $u_{ss_i} = \sum_j u_{s_i j}$, and $u_{t_i t} = \sum_j u_{j t_i}$.