Network Optimization (Fall 2008) – Solutions to Practice Midterm

2. Paths: P_1 = 1-3-5, $\Delta(P_1) = 2$; P_2 = 1-2-5, $\Delta(P_2) = 1$; P_3 = 1-2-5-4, $\Delta(P_3) = 2$;
   Cycles: W_1 = 2-5-4-2, $\Delta(W_1) = 3$; W_2 = 2-3-5-4-2, $\Delta(W_2) = 2$.
   The decomposition is not unique.

3. `pred` = [0 3 1 3 4 5]; longest path distance labels $d(\cdot) = [0\ \ -4\ \ -8\ \ -7\ \ -2\ \ -3]$.

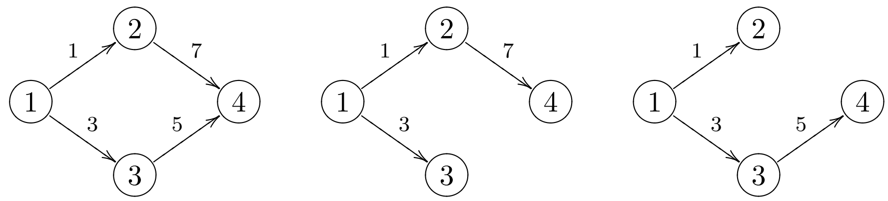
   The easiest way to solve this problem is probably by eye-balling! Dijkstra's algorithm does not apply here (due to the negative costs). At the same time, after guessing the solution, you could check the optimality conditions (although, we are not covering Chapter 5 in the midterm exam!).

4. Use the capacity-removal transformation to remove all u_{ij}'s, and at the same time, create a bipartite structure. If you use the transformation discussed in class (add a supply node with supply of u_{ij} for each arc (i, j), replace $b(i)$ with $b(i) - u_{ij}$), you can take as source nodes for the transportation problem the arcs of the original network (i.e., the extra nodes with supplies of u_{ij} added for each arc (i, j)). The original nodes of the network become the sink (demand) nodes. For each original node i, the new demand will be given by $b'(i) = b(i) - \sum_{(i,j)\in A} u_{ij} \leq 0$ by the assumption on supply nodes and capacities of arcs coming into it. You could also use the capacity removal transformation given in the book, in which case the nodes added for each arc will form the demand nodes in the transportation problem.

5. (a) The answer is `iv` (none of the above). There is no relationship between the number of arcs in the depth-first path and the shortest path from the source node to any other node.
   (b) The decomposition of a flow in arcs to flows in paths and cycles can be unique depending on the network in question (`iv`). For example, consider the case where the original flow is just along a cycle, and there are no other cycles in the network.

6. (a) FALSE. $f(n) + g(n) \neq O(\min(f(n), g(n)))$, as seen by the counterexample with $f(n) = n^2$ and $g(n) = n$.
   (b) TRUE. We examine the nodes in a topological order. For each node j examined, we set $p(j) = \displaystyle\sum_{(i,j)\in A} p(i)$.

7. Counterexample:



8. We can model the capacity expansion problem as a circulation problem on a network with nodes $N = \{s, 0, 1, \ldots, n\}$. There are two main sets of arcs – (s, i) for $i = 0, 1, \ldots, n$, and $(i, i+1)$ for $i = 0, 1, \ldots, n-1$. We also have the arc (n, s). We set $l_{s0} = 0, u_{s0} = \infty, c_{s0} = 0$; $l_{01} = x_0, u_{01} = x_0, c_{01} = 0$; $l_{si} = 0, u_{si} = v_{i-1}, c_{si} = a_{i-1}$ for $i = 1, \ldots, n$; $l_{i,i+1} = l_i, u_{i,i+1} = u_i, c_{i,i+1} = c_i$ for $i = 1, \ldots, n-1$; and $l_{ns} = l_n, u_{ns} = u_n, c_{ns} = c_n$. x_i is modeled as the flow in the arc $(i, i+1)$ and y_i is modeled as the flow in the arc $(s, i+1)$ for $i = 0, \ldots, n-1$. x_n is modeled as the flow in the arc (n, s). The flow balance equation for node $i+1$ is exactly $x_{i+1} = x_i + y_i$.

   In the figure, the numbers on arc (i, j) are $[l_{ij}, u_{ij}, c_{ij}]$. Notice that node 0 is added just to avoid parallel arcs going from node s to node 1.