REVIEW OF *A FIRST COURSE IN LINEAR ALGEBRA* BY ROBERT A. BEEZER

DAVID S. WATKINS*

As textbook prices head into the stratosphere, it is gratifying to observe that some people are trying to do something about it. Robert Beezer has made his text *A First Course in Linear Algebra* available to the public for free. A MathML hypertext version is on the web at http://linear.ups.edu, you can download a pdf version, or you can buy an inexpensive paper copy through the print-on-demand service lulu.com. The online and pdf versions contain links back to all theorems, definitions, etc., as they are used.

This book is a work in progress. There are three parts: Core, Topics, and Applications, but only the Core part resembles a finished book. The other two are just beginning to take form. The author envisions adding more and more to the book over time, and he encourages all of us to help out by making contributions. To this end he has made the TEX source code available. Users of the book are free to modify it and share their modifications with others, subject to the terms of the GNU Free Document License. As the author notes in the preface, "[This book] will never go 'out of print' nor will there ever be trivial updates designed to frustrate the used book market."

So what is the book like? First of all, all chapters, sections, theorems, and other items are given acronyms instead of numbers. Some examples: The chapter on vector spaces is Chapter VS. A theorem that says that linear transformations take zero to zero is called Theorem LTTZZ. The section on injective linear transformations is Section ILT. This practice has the advantage that Theorem LTTZZ will always have that name, even as the book changes shape over time. The disadvantage is that the text is cluttered with acronyms, most of which will have no immediate meaning for the reader.

The focus of the book is theoretical. Quoting again from the preface: "This textbook is designed to teach the university mathematics student the basics of the subject of linear algebra and the techniques of formal mathematics." Thus the book is meant for mathematics majors, and proofs and proof techniques are emphasized. The tone of the book is mostly formal; equations are preferred over words. Most of the proofs contain long chains of equalities like this one taken from the proof of Theorem OSIS (one side is sufficient), which shows that a right inverse is also a left inverse:

$$\begin{aligned}
BA &= (BA)I_n && \text{Theorem MMIM} \\
&= (BA)(BC) && \text{Theorem CINM} \\
&= B(AB)C && \text{Theorem MMA} \\
&= BI_nC && \text{Hypothesis} \\
&= BC && \text{Theorem MMIM} \\
&= I_n && \text{Theorem CINM}
\end{aligned}$$

In the electronic versions, the cited theorems have links back to their statements. In the paper version, page numbers are given.

[1]Department of Mathematics, Washington State University, Pullman, Washington 99164-3113. E-mail: watkins@wsu.edu.

This is a text for an *algebra* course. As the author states in the very first section, "...we will maintain an *algebraic* approach to the subject, with the geometry being secondary. ...here and now we are laying our bias bare." Indeed, the book is entirely algebraic; geometry is nowhere to be found.

The book begins concretely. There is a chapter on solving linear systems using row-echelon form. This is followed by a chapter on vectors in $\mathbb{C}^n$ that introduces concepts such as linear combinations, linear independence, spanning sets, and orthogonality. Then a chapter on matrices introduces matrix operations, the matrix inverse, null space, row space, column space, and the like. The relationships between these concepts and the solution of linear systems are laid out. After this the book takes an abstract turn, with chapters on vector spaces and linear transformations. Sandwiched between these are chapters on determinants and eigenvalues of matrices. This ordering strikes me as odd, but perhaps the order is not so important. The final chapter of the Core part of the book is on representations of linear transformations by matrices, culminating in the Jordan canonical form.

In my paper copy of the book, the Topics and Applications parts are empty. On the web there are, as of this writing, a couple of sections in Applications and ten or so in Topics. The author's intent is that the Topics part will include topics that the author deems not to be part of the core but that some instructors might like to include in their courses. For example, the fact that Gaussian elimination yields an *LU* decomposition of a matrix is included in Topics and not mentioned in the Core.

This choice reminds us once again of the bias of the book. This is definitely a "pure math" treatment of linear algebra. It is assumed that all arithmetic can be done exactly. Nowhere is it mentioned that the principal algorithms presented in the Core break down when executed in floating-point arithmetic on a computer or calculator. (For example, row reduction cannot determine reliably whether a square matrix is singular or not.) Condition numbers are not mentioned, nor are the superior numerical properties of unitary transformations. The Gram-Schmidt process is introduced in the core, but the *QR* decomposition is not mentioned at all. (Hopefully it will at least be added as a topic later.) The singular value decomposition is included as a mere topic. We are told that it is useful, but no specific uses (e.g. reliable rank determination) are given. Of course, all of this can be changed in the future.

Here's one other thing I would change: The index is way too long for a single web page. It ought to be split over many pages, perhaps 26 or so.

The preference for symbols over words sometimes has bad consequences. For example, at the beginning of Chapter D (determinants) a square matrix $E_{i,j}$ is defined as follows:

$$[E_{i,j}]_{kl} = \begin{cases} 0 & k \neq i,\ k \neq j,\ l \neq k \\ 1 & k \neq i,\ k \neq j,\ l = k \\ 0 & k = i,\ l \neq j \\ 1 & k = i,\ l = j \\ 0 & k = j,\ l \neq i \\ 1 & k = j,\ l = i \end{cases}$$

Got that? Here's what I would have preferred: "$E_{i,j}$ is the matrix obtained by interchanging the ith and jth rows of the identity matrix."

I was amused by Example CVS (crazy vector space). This is the set of all ordered pairs of complex numbers with the operations

$$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1 + 1, x_2 + y_2 + 1)$$

$$\alpha(x_1, x_2) = (\alpha x_1 + \alpha - 1, \alpha x_2 + \alpha - 1).$$

This really is a vector space! (What is the "zero" element?) It is a wonderful example in the spirit of abstract algebra, but there is no way I would show it to my sophomores. I want to develop their intuition, not crush it.

Would I use this book in my course? For me the total omission of geometry disqualifies it immediately. A *first* course in linear algebra ought to contain lots of geometry and pictures (along with the algebra) to help those sophomores develop some insight. Moreover, at my institution the first linear algebra course is populated mainly by engineering and science majors. They need to get the basics; they do not need an introduction to formal mathematics. That is left for other courses, including the second linear algebra course, which is mainly for mathematics majors. Beezer's book would surely be a useful supplemental resource for that course.

Clearly Beezer and I have some philosophical differences about what ought to be in a first course in linear algebra. Nevertheless I commend him for undertaking this project. Instructors who wish to teach a pure linear algebra course that emphasizes rigor and formal mathematics will be able to make good use of this material and feel secure in the knowledge that the book is not going to go out of print. Finally, the price is right.