Consider now the jth entry of Av_+ and let $p_1(j)$ be as defined in equation (2.7). If $p_1(j) = 0$ then $(Ax_+(t))_j = 0$. If $1 \leq p_1(j) \leq p_1 - 1$, then on examining the asymptotic effect of (2.8) in (3.11), we obtain that there exists a sufficiently large $t_0(j) \geq 0$ such that

$$(Ax_+(t))_j > 0, \qquad \forall t \geq t_0(j) \tag{3.12}$$

or equivalently,

$$Av_+ \in X_A(R^n_+). \tag{3.13}$$

The proof that $A^m x_+ \in X_A(R^n_+)$ for any $m \geq 2$ follows similarly.

(iii) ⇒(i) On letting $m = p_1 + 1$, this implication is trivial. ■

Acknowledgment

The authors would like to thank Professor Ronald J. Stern for very helpful discussions.

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