

Name: _____

Suppose that f_1, f_2, \dots, f_m are convex functions from \mathbb{R}^n into \mathbb{R} and let $f(x) = \sum_{k=1}^m f_k(x)$. Show that f is convex.

Solution. A function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is convex if for each $x, y \in \mathbb{R}^n$ and each $0 \leq a \leq 1$, $f(ax + (1 - a)y) \leq af(x) + (1 - a)f(y)$. Note:

$$\begin{aligned} f(ax + (1 - a)y) &= \sum_{k=1}^m f_k(ax + (1 - a)y) && \text{[def. of } f\text{]} \\ &\leq \sum_{k=1}^m (af_k(x) + (1 - a)f_k(y)) && \text{[} f_k \text{ are convex]} \\ &= a \sum_{k=1}^m f_k(x) + (1 - a) \sum_{k=1}^m f_k(y) && \text{[summation is linear]} \\ &= af(x) + (1 - a)f(y) && \text{[def. of } f\text{].} \end{aligned}$$