Job Sequencing with Delays

A facility needs to perform several jobs. The jobs must be performed in series, but can be performed in any order. Between jobs there may or may not be a delay due to the transfer of skilled personnel into (and out of) the facility. These delays are because the facility has limited space and has security check requirements. There are \( p \) skilled personnel and \( m \) jobs to complete. We also know the subset of people needed to complete job \( i \): \( S_i \subseteq \{1, 2, \ldots, p\} \). The facility can hold up to \( P \) personnel at one time. Each transfer of a person into or out of the facility requires time \( T \) to complete. How should the facility manager order the jobs in order to minimize the total delay times due to personnel movement?

Solution. We need to determine the order of jobs and understand when it is necessary to move personnel into or out of the facility. We will construct an integer program to solve this problem. We will use the following binary decision variables.

\[
x_{ik} = \begin{cases} 
1 & \text{if job } i \text{ is in the } k^{th} \text{ time slot} \\
0 & \text{otherwise}
\end{cases}
\]

\[
y_{jk} = \begin{cases} 
1 & \text{if person } j \text{ is in the facility during the } k^{th} \text{ time slot} \\
0 & \text{otherwise}
\end{cases}
\]

We assume that no personnel are in the facility at the beginning of the day and define \( y_{j0} = 0 \) for all \( j \). We also require that the work day end with all personnel processed out of the facility and define \( y_{jm+1} = 0 \). The objective is to minimize the total time delay in transfers. A delay occurs for each person which must leave the facility or enter the facility. That is

\[
\min f(x, y) = \sum_{j=1}^{p} \sum_{k=1}^{m+1} |y_{jk} - y_{jk-1}|.
\]

There are a variety of constraints. First, we ensure that each job is completed somewhere in the sequence:

\[
\sum_{k=1}^{m} x_{ik} = 1, \quad i = 1, 2, \ldots, m.
\]

We also make sure that exactly one job is active during each time slot:

\[
\sum_{i=1}^{m} x_{ik} = 1, \quad k = 1, 2, \ldots, m.
\]

The facility can hold a limited number of people during any time slot:

\[
\sum_{j=1}^{p} y_{jk} \leq P, \quad k = 1, 2, \ldots, m.
\]

Finally, we require that the correct skilled personnel are on hand for each job at the right time:

\[
x_{ik} \leq y_{jk} \quad \forall j \in S_i, \quad i, k = 1, 2, \ldots, m.
\]

Notice that as long as the capacity of the facility (\( P \)) is not exceeded, personnel are allowed to remain in the facility if they will be needed for another job later in time.