Math 464 QUIZ March 22 Name: Solution

Consider the linear program

$$\begin{array}{lll} \min & z = x_1 + 2x_2 + 3x_3 + 4x_4 & \\ \text{s.t} & Ax \geq b & \text{(LP)} \\ & x \geq 0 & \end{array}$$

where $b = (4\,3\,2\,1)^T$ and A is a 4×4 real matrix. Suppose $x = (2\,0\,1\,1)^T$ is a feasible point for (LP). Explain why $y = (1\ 1\ 1\ 1)^T$ cannot be a feasible point for the corresponding dual problem.

We know that if x and y are primal and dual feasible, respectively, then $c^T x \geq b^T y$. Note:

$$c^T x = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix} = 2+0+3+4 = 9,$$

$$b^T y = \begin{bmatrix} 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 4+3+2+1 = 10.$$

Since x is primal feasible and $b^T y > c^T x$, y cannot be dual feasible.