Math 464 QUIZ March 22 Name: Solution

Consider the standard form linear program

$$\begin{aligned} \min \quad & z = c^T x \\ \text{s.t} \quad & Ax = b \qquad \text{(P)} \\ & x \geq 0 \end{aligned}$$

and its dual linear proram

$$\begin{aligned} \max \quad & w = b^T y \\ \text{s.t} \quad & A^T y \leq c \qquad \text{(D)} \end{aligned}$$

Show that a (possibly infeasible) basis of (P) with non-negative reduced costs ($\bar{c}^T = c^T - c_B^T A_B^{-1} A$) corresponds to a feasible solution $y^T = c_B^T A_B^{-1}$ of (D). Show that if the basis is also primal feasible, then the optimal objective costs are equal.

(a) We show that $\bar{c}^T \geq 0 \Rightarrow A^T y \leq c$.

$0 \leq \bar{c}^T = c^T - c_B^T A_B^{-1} A = c^T - y^T A \Rightarrow c^T \geq y^T A$ or $A^T y \leq c$.

(b) If $B$ is primal feasible, then $x_B \geq 0$, and if $\bar{c}^T \geq 0$ then $B$ is primal optimal with objective $c^T x = c_B^T x_B = c_B^T A_B^{-1} b = y^T b = b^T y$. Since $b^T y \leq c^T x$ for all feasible $x$ and $y$, ~~the~~ $x$ and $y$ are primal and dual optimal, respectively, and the optimal costs are equal.