Claim: The intersection of a finite number of polyhedra is a polyhedron.

Proof. Suppose $P_1, P_2, \ldots, P_m$ are $m$ polyhedra in $\mathbb{R}^n$. We demonstrate that $P = \bigcap_{k=1}^{m} P_k$ is a polyhedron by showing that it can be written as the intersection of a finite number of halfspaces.

Each polyhedron $P_k = \{x \in \mathbb{R}^n \mid A_k x \geq b_k\}$ for some $q_k \times n$ matrix $A_k$ and some $b_k \in \mathbb{R}^{q_k}$. The intersection is $P = \{x \in \mathbb{R}^n \mid A_1 x \geq b_1, A_2 x \geq b_2, \ldots, A_m x \geq b_m\}$, which is the intersection of $q = \sum_{k=1}^{m} q_k$ halfspaces. Thus, $P$ is a polyhedron. ☐