

Math 464 Homework problems #8 and #9

Let $\Omega \subseteq \mathbb{R}^n$. Suppose $f : \Omega \rightarrow \mathbb{R}$ is a function and for each $c \in \mathbb{R}$, define the sets $S_c = \{x \in \Omega \mid f(x) \leq c\}$.

(8) (a) Test the following statement with an example where $n \geq 2$.

If f is convex then $S_c \subseteq S_d$ whenever $c \leq d$. (A)

(b) Prove statement (A).

(9) (a) State the converse of statement (A). Given an example showing that the converse is false.

(b) Salvage the converse by adjusting (or adding to) the hypothesis to make a true statement, but without changing the flavor too much.

(c) Prove the salvaged statement.