Q: Determine whether or not the following transformation is linear.

$T: \mathbb{R}^2 \to \mathbb{R}$ defined by $T\begin{pmatrix} a \\ b \end{pmatrix} = a+b+3$.

A: According to Theorem ___, if T does not map the zero vector in $\mathbb{R}^2$ to the zero vector in $\mathbb{R}$, then T is not linear. Notice: $T\begin{pmatrix} 0 \\ 0 \end{pmatrix} = 3 \neq 0$ so T is not linear.

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$T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ a+b \end{pmatrix}$.

A: To show that T is linear consider arbitrary vectors $v = \begin{pmatrix} a \\ b \end{pmatrix}$ and $w = \begin{pmatrix} c \\ d \end{pmatrix}$ and arbitrary scalar e, if $T(ev+w) = eT(v)+T(w)$, then T is linear.

$$T(ev+w) = T\begin{pmatrix} ea+c \\ eb+d \end{pmatrix}$$

$$= \begin{pmatrix} ea+c \\ eb+d+ea+c \end{pmatrix}$$

$$= e\begin{pmatrix} a \\ a+b \end{pmatrix} + \begin{pmatrix} c \\ c+d \end{pmatrix}$$

$$= eT(v) + T(w).$$

T is linear.