NAME:

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MATH 420 – Quiz #1

Write a **formal** proof of the following statement.

> Let $(V, +, \cdot)$ be a vector space over F, then $0 \cdot x = 0$ for each $x \in V$.

You may reference the following theorem in your proof.

> **Theorem 3.4.1** If x, y, and z are vectors in a vector space $(V, +, \cdot)$ and $x + z = y + z$, then $x = y$.

You may also reference the vector space properties (written on the board).

Solution:

Proof. We show that for arbitrary $x \in V$, $0 \cdot x = 0$ by using the vector space properties in the definition of a vector space.

$$\begin{aligned} 0 \cdot x + x &= 0 \cdot x + 1 \cdot x && \text{(P10)} \\ &= (0+1) \cdot x && \text{(P7)} \\ &= 1 \cdot x \\ &= x && \text{(P10)} \\ &= 0 + x && \text{(P8)} \end{aligned}$$

Thus, by Theorem 3.4.1, $0 \cdot x = 0$. □