

Name: _____

WSU ID#: _____

MATH 420 Fall 2018

Midterm Exam #1

Instructions:

1. Write your name and WSU ID# at the top of this page.
2. Your work is to be your own work, unassisted by any other person or their work.
3. Do not use any materials other than those provided with this exam.
4. Do not include scratchwork. Please use the supplied blank paper to formulate your thoughts.
5. Always justify your work. Always explain linear algebra concepts central to the question.
6. This exam includes all definitions and theorems which you may require.
7. There are six questions.
8. The exam is worth 70 points. The first five questions are worth 10 points each. The last question has multiple parts and is worth 20 points.
9. You must use appropriate mathematical notation and language to receive full credit.

1	2	3	4	5	6	
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Definition 1. Consider a set of objects X with scalars taken from \mathbb{R} , and operations of addition (+) and scalar multiplication (\cdot) defined on X . We say that X is closed under addition if $x + y \in X$ for all $x, y \in X$. We say that X is closed under scalar multiplication if $\alpha \cdot x \in X$ for each $x \in X$ and each $\alpha \in \mathbb{R}$.

Definition 2. Consider a set V over a field \mathbb{F} (either \mathbb{R} or Z_2) with given definitions for addition (+) and scalar multiplication (\cdot). V with + and \cdot is called a vector space over \mathbb{F} if for all $u, v, w \in V$ and for all $\alpha, \beta \in \mathbb{F}$, the following ten properties hold.

(P1) $u + v \in V$.

(P2) $\alpha \cdot v \in V$.

(P3) $u + v = v + u$.

(P4) $(u + v) + w = u + (v + w)$.

(P5) $\alpha \cdot (\beta \cdot v) = (\alpha\beta) \cdot v$.

(P6) $\alpha \cdot (u + v) = \alpha \cdot u + \alpha \cdot v$.

(P7) $(\alpha + \beta) \cdot v = \alpha \cdot v + \beta \cdot v$.

(P8) There exists $0 \in V$, called the additive identity, such that $0 + v = v + 0 = v$.

(P9) There exists $z \in V$, such that $v + z = 0$, called the additive inverse of v .

(P10) The scalar set \mathbb{F} has an identity element, denoted 1, for scalar multiplication:
 $1 \cdot v = v$.

Theorem 1. If x, y , and z are vectors in a vector space $(V, +, \cdot)$ and $x + z = y + z$, then $x = y$.

Theorem 2. Let $(V, +, \cdot)$ be a vector space over \mathbb{F} . $0 \cdot x = 0$ for each $x \in V$.

Theorem 3. Let $(V, +, \cdot)$ be a vector space over \mathbb{R} with additive identity element 0. $\alpha \cdot 0 = 0$ for each $\alpha \in \mathbb{R}$.

Theorem 4. Let $(V, +, \cdot)$ be a vector space over \mathbb{F} . $(-\alpha) \cdot x = -(\alpha \cdot x) = \alpha \cdot (-x)$ for each $\alpha \in \mathbb{F}$ and each $x \in V$.

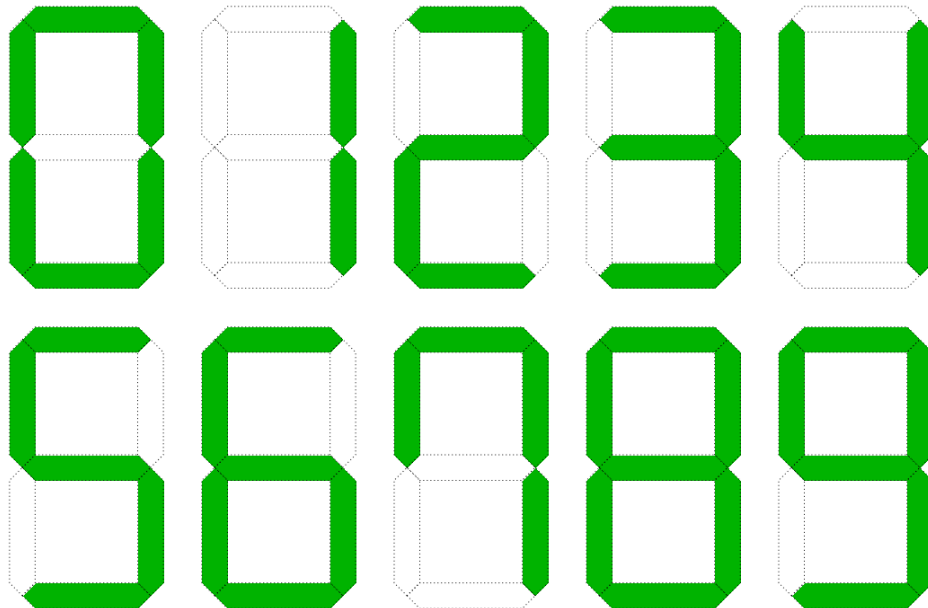


Figure 1: The ten digits of a standard 7-bar LCD display, as vectors in $\mathcal{D}(Z_2)$, where Z_2 is the field that includes only the scalar values 0 and 1.

Definition 3. Let $(V, +, \cdot)$ be a vector space over a field \mathbb{F} . If $W \subseteq V$ (W is a subset of V), then we say that W is a subspace of $(V, +, \cdot)$ whenever $(W, +, \cdot)$ is also a vector space.

Theorem 5. Let $(V, +, \cdot)$ be a vector space over a field \mathbb{F} . Then $W \subseteq V$ (W is a subset of V) if and only if $(W, +, \cdot)$ is a subspace of V if and only if $0 \in W$ and W is closed under (the inherited operations of) vector addition and scalar multiplication.

Theorem 6. Let W_1 and W_2 be any two subspaces of a vector space $(V, +, \cdot)$. Then $W_1 \cap W_2$ is a subspace of $(V, +, \cdot)$.

Theorem 7. Let W_1 and W_2 be any two subspaces of a vector space V . Then $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.

Theorem 8. Let W_1 and W_2 be subspaces of a vector space $(V, +, \cdot)$. Then $W_1 + W_2$ is a subspace of $(V, +, \cdot)$.

Definition 4. Let $(V, +, \cdot)$ be a vector space over \mathbb{F} . Given a finite set of vectors $v_1, v_2, \dots, v_k \in V$, we say that the vector $w \in V$ is a linear combination of v_1, v_2, \dots, v_k if $w = a_1v_1 + a_2v_2 + \dots + a_kv_k$ for some scalar coefficients a_1, a_2, \dots, a_k .

Definition 5. Let $(V, +, \cdot)$ be a vector space and let $X \subseteq V$. The span of the set X , denoted **span** X , is the set of all linear combinations of the elements of X . In addition, **span** $\emptyset \equiv \{0\}$.

Definition 6. We say that the (possibly infinite) set S spans (or generates) a set W if $W = \mathbf{span} S$. In this case, we also say that the vectors in S span the set W .

Definition 7. We say that a (possibly infinite) set S is a spanning set (or generating set) for W if $W = \mathbf{span} S$.

Theorem 9. Let X be a subset of vector space $(V, +, \cdot)$. Then **span** X is a subspace of V .

Theorem 10. Let X and Y be subsets of a vector space $(V, +, \cdot)$. Then the following statements hold.

- (a) **span** $(X \cap Y) \subseteq (\mathbf{span} X) \cap (\mathbf{span} Y)$.
- (b) **span** $(X \cup Y) \supseteq (\mathbf{span} X) \cup (\mathbf{span} Y)$.
- (c) If $X \subseteq Y$, then **span** $X \subseteq \mathbf{span} Y$.
- (d) **span** $(X \cup Y) = (\mathbf{span} X) + (\mathbf{span} Y)$.