MATH 420 – Chapter 4, Supplementary Homework
Including notes from class

For **three** of the sets given below, respond to the following questions.

a Give two example elements of the set.

b Define the set mathematically using set notation.

c Provide natural definitions for the operations of addition (+) and scalar multiplication ($\cdot$).

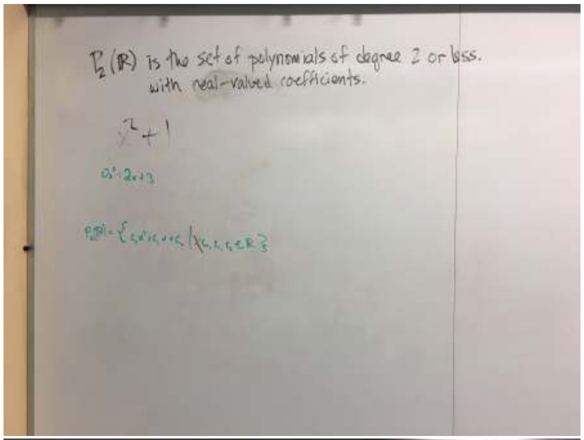
d Is the given set, along with + and $\cdot$ a vector space? Justify.

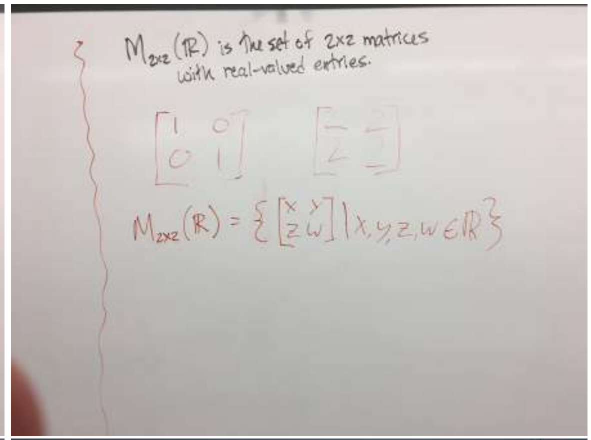
e Find a proper (nontrivial) subset of the set which is also a vector space. Justify.

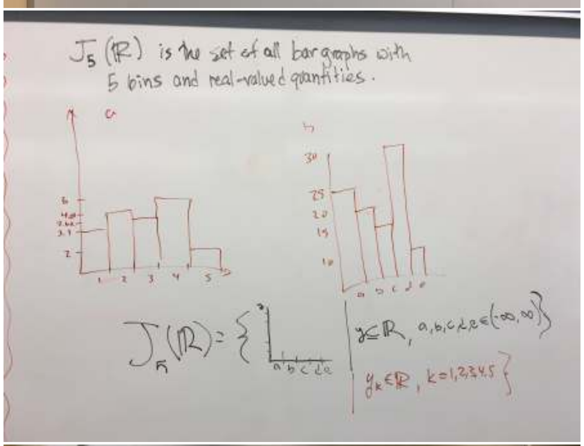
f Find a proper subset of the set which is not a vector space. Justify

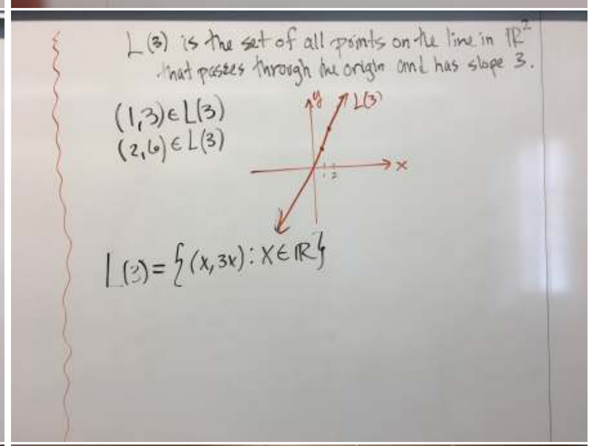
Here are the sets:

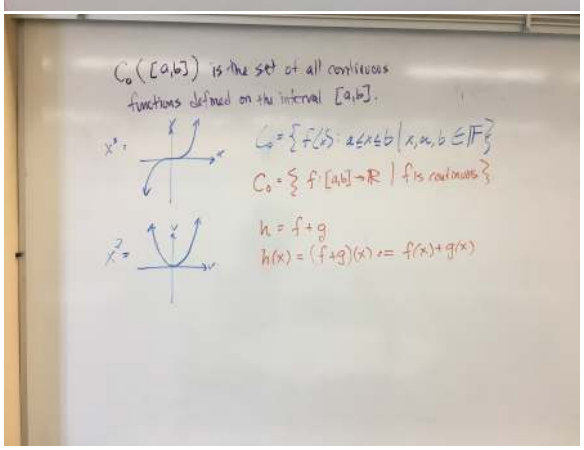
1. $\mathcal{P}_2(\mathbb{R})$ is the set of all polynomials of degree 2 or less with real-valued coefficients.
2. $\mathcal{M}_{2\times 2}(\mathbb{R})$ is the set of 2×2 matrices with real-valued entries.
3. $\mathcal{J}_5(\mathbb{R})$ is the set of all bar graphs with 5 bins and real-valued quantities.
4. $\mathcal{L}(3)$ is the set of all points on the line in $\mathbb{R}^2$ that passes through the origin and has slope 3.
5. $\mathcal{C}_0([a, b])$ is the set of all continuous functions defined on the real interval $[a, b]$.
6. $\mathcal{S}(\mathbb{R})$ is the set of all infinite sequences of real numbers that have a finite number of nonzero entries.

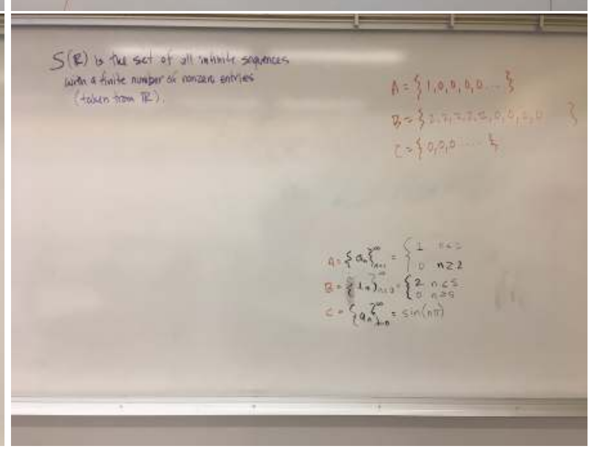

P2(R) is the set of polynomials of degree 2 or less.
with real-valued coefficients.
x2+1


M2x2(R) is the set of 2x2 matrices
with real-valued entries.
M2x2(R) = { [x y; z w] | x, y, z, w ∈ R }


J5(R) is the set of all bar graphs with
5 bins and real-valued quantities.
a, b, c, d, e ∈ (-∞, ∞)
yk ∈ R, k = 1,2,3,4,5


L(3) is the set of all points on the line in R2
that passes through the origin and has slope 3.
(1,3) ∈ L(3)
(2,6) ∈ L(3)
y
L(3)
x
L(3) = {(x, 3x) : x ∈ R}


C0([a,b]) is the set of all continuous
functions defined on the interval [a,b].
x3 =
x2 =
C0 = { f : [a,b] → R | f is continuous }
h = f + g
h(x) = (f+g)(x) := f(x) + g(x)


S(R) is the set of all infinite sequences
with a finite number of nonzero entries
(taken from R).
A = {1, 0, 0, 0, 0, ...}