

MATH 420 – Chapter 4, Supplementary Homework

For **three** of the sets given below, respond to the following questions.

- a Give two example elements of the set.
- b Define the set mathematically using set notation.
- c Provide natural definitions for the operations of addition (+) and scalar multiplication (\cdot).
- d Is the given set, along with + and \cdot a vector space? Justify.
- e Find a proper (nontrivial) subset of the set which is also a vector space. Justify.
- f Find a proper subset of the set which is not a vector space. Justify

Here are the sets:

1. $\mathcal{P}_2(\mathbb{R})$ is the set of all polynomials of degree 2 or less with real-valued coefficients.
2. $\mathcal{M}_{2 \times 2}(\mathbb{R})$ is the set of 2×2 matrices with real-valued entries.
3. $\mathcal{J}_5(\mathbb{R})$ is the set of all bar graphs with 5 bins and real-valued quantities.
4. $\mathcal{L}(3)$ is the set of all points on the line in \mathbb{R}^2 that passes through the origin and has slope 3.
5. $\mathcal{C}_0([a, b])$ is the set of all continuous functions defined on the real interval $[a, b]$.
6. $\mathcal{S}(\mathbb{R})$ is the set of all infinite sequences of real numbers that have a finite number of nonzero entries.