Principles of Optimization – Standard Form

1. Consider the Farmer problem again:
   A farmer has 100 acres of land and 560 hours of time to work with the crops on that land. Two types of crops can be planted: soy and oats. Soy requires 5 hours of labor per acre and oats 7. Soy yields 10 bushels per acre and oats 20. The profit per bushel harvested is \$3 for soy and \$4 for oats. A government regulation requires that at least 270 bushels of soy be produced. How many acres should be planted in each crop in order to maximize profit?
   We modeled this problem as follows:
   Let x_1 be the number of acres of soy to plant and x_2 be the number of acres of oats to plant. We want to maximize the farmer's profit (z, in dollars).

$$\boxed{\begin{array}{ll} \min\limits_{x \in \mathbb{R}^2} & -z = -30x_1 - 80x_2 \\ \text{s.t.} & x_1 + x_2 \leq 100 \\ & 5x_1 + 7x_2 \leq 560 \\ & x_1 \geq 27 \\ & x \geq 0 \end{array}}$$

   Standard form of a linear program is

$$\boxed{\begin{array}{ll} \min\limits_{x} & z = c^T x \\ \text{s.t.} & Ax = b \\ & x \geq 0 \\ & x \in \mathbb{R}^n \end{array}}$$

   Do you believe it possible to write the farmer's linear program in standard form? Explain your reasoning.

   (many possible responses)

2. Reformulate this problem in terms of the original decision variables and, in addition, keeps track of the unused acres.

Let x_3 = # of acres not to plant

Then we have $x_3 \geq 0$ and the acres constraint is now an equality constraint:

$$x_1 + x_2 + x_3 = 100 .$$

we have:

$$\boxed{\begin{array}{ll} \min\limits_{x \in \mathbb{R}^3} & (-z) = -30x_1 - 80x_2 \\ \text{s.t.} & x_1 + x_2 + x_3 = 100 \\ & 5x_1 + 7x_2 \leq 560 \\ & x_1 \geq 27 \\ & x \geq 0 \end{array}}$$

3. Reformulate the entire problem in standard form. Be sure to interpret any new decision variables.

let x_4 = # of unused labor hours.

x_5 = # of acres planted in soy above 27.

Then we have the standard form problem

$$\boxed{\begin{aligned} \min_{x \in \mathbb{R}^5} \quad & (-z) = -30x_1 - 80x_2 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 = 100 \\ & 5x_1 + 7x_2 + x_4 = 560 \\ & x_1 \qquad - x_4 = 27 \\ & x \geq 0 \end{aligned}}$$

or, in standard (matrix) form

$$\boxed{\begin{aligned} \min_{x \in \mathbb{R}^5} \quad & (-z) = c^T x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \\ & c^T = [-30 \ -80 \ 0 \ 0 \ 0] \\ & A = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 7 & 1 \\ 1 & 0 & -1 \end{bmatrix}, \ b = \begin{bmatrix} 100 \\ 560 \\ 27 \end{bmatrix} \end{aligned}}$$