# Math 364 – A First Example – Continued

In the farmer example problem we found that the highest-profit planting strategy would be a solution to this problem:

$$
\begin{aligned}
\max_{x \in \mathbb{R}^2} \quad & z = 30x_1 + 80x_2 \\
\text{s.t.} \quad & x_1 + x_2 \leq 100 \\
& 5x_1 + 7x_2 \leq 560 \\
& x_1 \geq 27 \\
& x \geq 0
\end{aligned}
$$

Is this problem an example of the general optimization problem given in class (and shown below)? Be prepared to justify and defend your reasoning.

$$
\begin{aligned}
\max_{x \in \mathbb{R}^n} \quad & z = f(x) \\
\text{s.t.} \quad & g(x) \leq 0 \\
& h(x) = 0 \\
& x_k \in \mathbb{R} (\text{or } \mathbb{Z}), k = 1, 2, \cdots, n.
\end{aligned}
$$

Yes it is. To show this we can define the various components of the general form to produce the specific form:

$n=2$ $\quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\quad f(x) = 30x_1 + 80x_2$ $\quad h(x) = 0$

$$g(x) = \begin{bmatrix} x_1 + x_2 - 100 \\ 5x_1 + 7x_2 - 560 \\ 27 - x_1 \\ -x_1 \\ -x_2 \end{bmatrix} \qquad x \in \mathbb{R}^2$$

Convert the example optimization problem into the following form:

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & z = c^T x \\ \text{s.t.} \quad & Ax \le b \\ & \bar{A}x = \bar{b} \\ & \ell \le x \le u \end{aligned}$$

where $c, b, \bar{b}, \ell, u$ are vectors and $A, \bar{A}$ are matrices.

We need to define the various vector/matrix objects to produce the farmer problem. One solution is:

$$c = \begin{bmatrix} -30 \\ -80 \end{bmatrix} \quad \left( \begin{array}{l} \text{note: we are minimizing } -z \\ \text{instead of maximizing } z. \end{array} \right)$$

$$A = \begin{bmatrix} 1 & 1 \\ 5 & 7 \\ -1 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 100 \\ 560 \\ -27 \end{bmatrix}$$

$$\ell = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

and $\bar{A}, \bar{b}, u$ are unused variables: There are no equality constraints and neither variable $x_1$ or $x_2$ is bounded above.