Name ANSWER KEY with expected level of detail

WSU ID# ______________________________

Math 364 Quiz – Week #3

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = e^{-xy}$. Respond to the following questions with appropriate mathematical and English language skills.

1. Find all stationary points of f.

   Stationary points of f are the solutions of $\nabla f(x, y) = 0$.

   $$\nabla f(x,y) = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{bmatrix} = \begin{bmatrix} -ye^{-xy} \\ -xe^{-xy} \end{bmatrix} \overset{set}{=} 0.$$

   The only solution is $x = y = 0$. Thus, $(x, y) = (0, 0)$ is the unique stationary point.

2. Classify each stationary point using an appropriate second derivative test.

   The eigenvalues of the Hessian matrix at a stationary point will help us classify the point.

   $$\nabla^2 f(x,y) = \begin{bmatrix} \partial^2 f/\partial x^2 & \partial^2 f/\partial x \partial y \\ \partial^2 f/\partial y \partial x & \partial^2 f/\partial y^2 \end{bmatrix} = \begin{bmatrix} y^2 e^{-xy} & (xy-1)e^{-xy} \\ (xy-1)e^{-xy} & x^2 e^{-xy} \end{bmatrix},$$

   $$\nabla^2 f(0,0) = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}.$$

   From the note below, the eigenvalues of $\nabla^2 f(0, 0)$ are $\lambda = \pm 1$. Because the eigenvalues are of mixed sign, the stationary point $(0, 0)$ is a saddle point.

Note: The eigenvalues of matrix $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ are $\lambda = a, b$. The eigenvalues of matrix $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}$ are $\lambda = \pm\sqrt{ab}$.