# Order of Convergence

1. When considering various techniques for finding an optimal solution, $x^*$ to

$$\min_{x \in \mathbb{R}^n} \quad z = f(x)$$

   we seek the "best" technique. Suppose we have a handful of techniques that converge to $x^*$, what criteria would you use to determine which technique is better than the another? (Make a list.)

2. In practice, optimization techniques take infinitely many steps to find $x^*$. Suppose we choose a strategy to find the optimal solution $x^*$, but we terminate to get an approximate solution at $x_k$. How close is our approximate solution to the optimal solution?

3. Given a sequence of iterates $(x_k) \subset \mathbb{R}^n$ that converges to $x^* \in \mathbb{R}^n$, the optimal solution. Explain, in words, what each of the following scenarios would say about the sequence, being sure to give a ranking for the strategy that might produce such iterates.

(Scenario 1) $\dfrac{||x_{k+1} - x^*||}{||x_k - x^*||} = 1$

(Scenario 2) $\dfrac{||x_{k+1} - x^*||}{||x_k - x^*||^2} = \delta, \text{ for } \delta < 1.$

(Scenario 3) $\dfrac{||x_{k+1} - x^*||}{||x_k - x^*||} = \delta, \text{ for } \delta \geq 1$

(Scenario 4) $\dfrac{||x_{k+1} - x^*||}{||x_k - x^*||} = \delta_k, \text{ for } \delta_k \to 0 \text{ as } k \to \infty$

(Scenario 5) $\dfrac{||x_{k+1} - x^*||}{||x_k - x^*||} = \delta, \text{ for } \delta < 1$

4. Using Newton's method on $f(x,y) = x^4 + y^4 + x^3 + xy - 2xy^2$, starting at $(1, -3/2)$ the iterates will converge to the stationary point $x^* = (0.6815, -0.9299)$, determine whether the order of convergence is linear, superlinear, or quadratic.

5. Using a gradient descent method on $f(x,y) = x^4 + y^4 + x^3 + xy - 2xy^2$, starting at $(1, -3/2)$ the iterates will converge to the stationary point $x^* = (0.6815, -0.9299)$, determine whether the order of convergence is linear, superlinear, or quadratic.