Exercise 6d. Solve the following optimization problem.

$$\max_{x,y \in \mathbb{R}} \quad z = f(x,y) = e^{-x^2-2y^2} \quad \text{subject to } 2x+y \geq 1.$$

start with the complete problem statement.

Solution. Begin by finding the feasible stationary points of $f(x,y)$. Stationary points are solutions to $\nabla f(x,y) = 0$ and feasible points satisfy $2x+y \geq 1$.

Explain your approach.

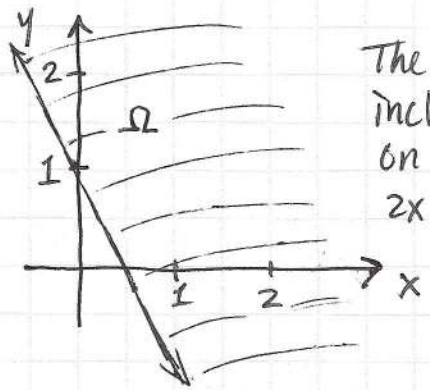
State the optimization concepts

$$\nabla f(x,y) = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{bmatrix} = \begin{bmatrix} -2x\, f(x,y) \\ -4y\, f(x,y) \end{bmatrix}$$

Notice that no scratchwork is included.

Since $f(x,y) \neq 0$ for any (x,y), the unique solution to $\nabla f = 0$ is $(x,y) = (0,0)$. However, this stationary point does not satisfy the constraint and is thus not feasible. So any optimal solution to the optimization problem must lie on the boundary of the feasible region.

State the interim conclusion then explain the optimization concepts that lead you to the next step.



The feasible region Ω includes all of $\mathbb{R}^2$ on and above the line $2x+y=1$.

This graph is optional, but it does clarify the details of the feasible region and its boundary.

Notice that as $x \to \pm\infty$ or $y \to \pm\infty$, $z \to 0$. We must also check the boundary $2x+y=1$. Along this line

$$f(x) = e^{-9x^2+8x-2}.$$

Again, no scratchwork.

We seek stationary points of this function as solutions to $f'(x) = 0$. If, at such a solution, $f''(x) < 0$ then x is a local maximizer.

Continue to explain optimization steps.

$$f'(x) = (-18x+8)f(x).$$

$$f''(x) = \left[-18 + (-18x+8)^2\right] f(x).$$

Notice that interim results necessary for understanding conclusions must be stated.

The unique stationary point of f is $x = 4/9$ at which $f''(x) < 0$. Thus, along with $y = 1 - 2x$, we have the optimal solution

$$\boxed{\begin{gathered}(x^*, y^*) = (4/9, 1/9) \\ z^* = f(x^*, y^*) = e^{-2/9}\end{gathered}}$$

Conclusions with no scratchwork.

Your final result is clearly indicated.

Values are never estimated using decimal numbers.

Notice that this solution can be easily read from start to finish. Use complete sentences, proper grammar, and no distracting scratchwork.

This solution demonstrates conceptual and computational knowledge of <u>optimization</u> concepts.