The next two pages illustrate appropriately written homework solutions. You should observe the following:

(1) Each problem begins on a new page.

(2) Each problem begins with a complete problem statement.

(3) The text is legible.

(4) The solutions are read from top to bottom, using standard English and mathematics.

(5) Scratchwork and algebraic details are absent.

(6) Solutions focus on optimization concepts.

(7) Optimization tests are stated, not assumed.

(8) Graphs are carefully drawn in a way that illustrates the relevant concepts and results.

(9) There is a conciseness to the solutions, but all essential results are justified.

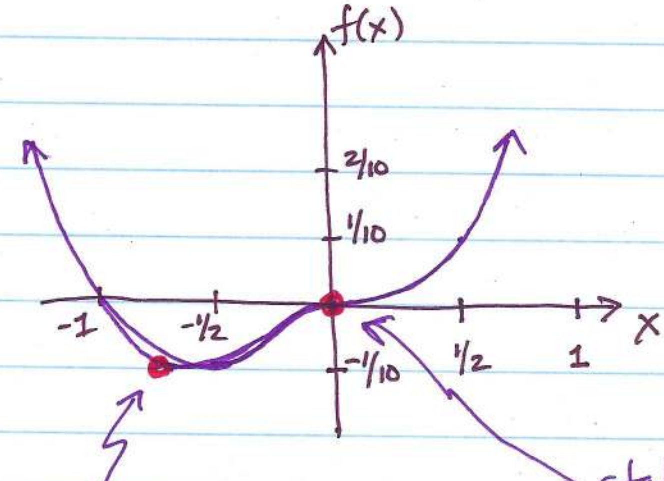
Example Homework Solutions
January 27, 2019

1. Show that the function $f(x) = x^4 + x^3$ has exactly two stationary points, one of which is a local minimizer and one of which cannot be classified by the second derivative test. Graph the function and describe the stationary points.

The stationary points of a function f are the solutions to $f'(x) = 0$. We have $f'(x) = 4x^3 + 3x^2 = x^2(4x+3)$. Thus, there are two stationary points: $x = 0$ and $x = -3/4$.

A stationary point x is a local minimizer if $f''(x) > 0$. We have $f''(x) = 12x^2 + 6x$, $f''(-3/4) = 9/4 > 0$, so $x = -3/4$ is a local minimizer.

A stationary point x cannot be classified by the second derivative test if $f''(x) = 0$. We have $f''(0) = 0$ so stationary point $x = 0$ cannot be classified by this test.



local minimizer at $x = -3/4$, $f(x) = -27/256$

stationary point at $x=0$, $f(x)=0$, that is neither a local max nor a local min.

2. Solve the following optimization problem:

min $z = f(x) = \ln(x^2+x+1)$ subject to $x \in \mathbb{R}$.

For an unconstrained problem, any minimizer must be a stationary point of the function; solutions to $f'(x)=0$. We have $f'(x) = \frac{2x+1}{x^2+x+1}$. ~~with~~ so $x = -1/2$ is the unique stationary point.

Notice that $f''(x) = \frac{-2x(x+1)}{(x^2+x+1)^2}$, $f''(-1/2) > 0$ so $x=-1/2$ is indeed a local minimizer.

$x=-1/2$ is the global minimizer if $\lim_{x\to\pm\infty} f(x) > f(-1/2)$.
Notice that $\lim_{x\to\pm\infty} f(x) = +\infty$. Thus, the solution:

$$\boxed{x^* = -1/2 \quad z^* = f(x^*) = \ln(3/4)}$$