

The Octave/Matlab Software form of a linear program is:

$$\begin{array}{l} \min_{x \in \mathbb{R}^n} z = c^T x \\ \text{s.t. } Ax \leq b \\ \bar{A}x = \bar{b} \\ l \leq x \leq u \end{array} \quad (1)$$

Show that the given form is an example of the general form of an optimization problem

$$\begin{array}{l} \max_{x \in \mathbb{R}^n} z = f(x) \\ \text{s.t. } g(x) \leq 0 \\ h(x) = 0 \end{array} \quad (2)$$

Solution: We must show that (1) can be written as (2) by providing appropriate definitions for $f(x)$, $g(x)$ and $h(x)$. First, noting that (1) is a minimization problem and (2) is a maximization problem, we have $f(x) = -c^T x$. Second, the equality constraints in (1) can be written $\bar{A}x - \bar{b} = 0$, so that we have $h(x) = \bar{A}x - \bar{b}$. Next, we consider all of the inequality constraints:

$$\left. \begin{array}{l} Ax \leq b \\ x \leq u \\ -x \leq -l \end{array} \right\} \Rightarrow \begin{bmatrix} A \\ I_n \\ -I_n \end{bmatrix} x \leq \begin{bmatrix} b \\ u \\ -l \end{bmatrix} \Rightarrow \begin{bmatrix} A \\ I_n \\ -I_n \end{bmatrix} x - \begin{bmatrix} b \\ u \\ -l \end{bmatrix} \leq 0$$

where A is an $m \times n$ matrix, I_n is the $n \times n$ identity matrix, and b, u, l are the given vectors in (1). We have

$$g(x) = \begin{bmatrix} A \\ I_n \\ -I_n \end{bmatrix} x - \begin{bmatrix} b \\ u \\ l \end{bmatrix}$$

Notice that $\begin{bmatrix} A \\ I_n \\ -I_n \end{bmatrix}$ is an $(m+2n)$ by n matrix and $\begin{bmatrix} b \\ u \\ l \end{bmatrix}$ is a $(m+2n)$ by 1 vector.

And $g: \mathbb{R}^n \rightarrow \mathbb{R}^{m+2n}$, $h: \mathbb{R}^n \rightarrow \mathbb{R}^m$ where \bar{A} is $m \times n$.