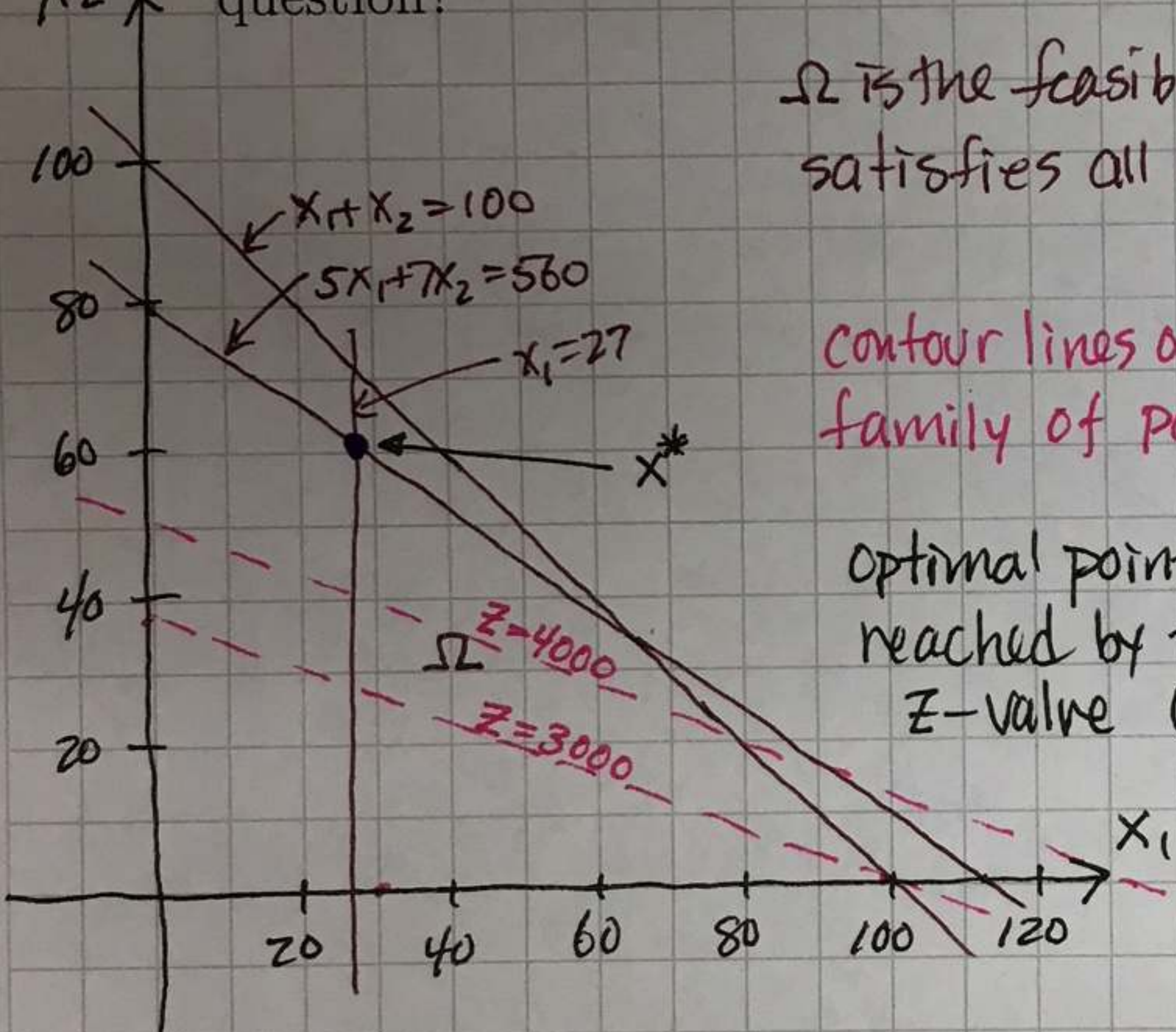
Again consider the farmer's optimization problem:

$$\max_{x \in \mathbb{R}^2} \quad \boxed{z = 30x_1 + 80x_2} \quad f(x_1, x_2)$$

$$\Omega \quad \text{s.t.} \quad \begin{array}{|l|} x_1 + x_2 \leq 100 \\ 5x_1 + 7x_2 \leq 560 \\ x_1 \geq 27 \\ x \geq 0 \end{array}$$

Sketch the feasible region in $\mathbb{R}^2$. Draw a few level lines (contours) of the objective function that also intersect the feasible region. How can you use this information to find an optimal solution to the farmer's question?



Ω is the feasible region – the set of points that satisfies all constraints (there are six!)

Contour lines of the objective function form a family of parallel lines.

Optimal points are those feasible points reached by the contour line of largest z-value (for a max problem).

We can use the above ideas to reach an important fact about linear programs:

Any optimal solution point must lie on the boundary of the feasible region and if an optimal solution exists then at least one optimal solution is a vertex of the feasible region*.

*unless the feasible region has no vertices