

All About the Gradient

For the following, consider the function $f(x, y) = x^2 + 2xy + y^2$.

1. State the domain space and the range space of f .
2. Accurately, sketch several level curves of f .
3. Find $\nabla f(x, y)$. In what space does $\nabla f(x, y)$ lie?

4. For each point (a, b) below, draw the level curves of f containing the point and draw $\nabla f(a, b)$

(a) $(1, 1)$

(b) $(-2, 0)$

(c) $(0, 3)$

Now, use $f(x, y) = (x - 2)^2 + (y + 2)^2$.

5. Find a vector parallel to the line tangent to the level curve of f at $(1, 1)$. Call this vector $T(1, 1)$.

6. Compute $\nabla f(1, 1)$ and determine how $T(1, 1)$ and $\nabla f(1, 1)$ are related.

7. Repeat for the points $(-2, 0)$ and $(0, 3)$.

8. What do you notice about the direction of ∇f in reference to the function f ?

9. What do you notice about ∇f as it relates to the level curves?

10. Using your result, suggest an algorithm to step through the domain space to find stationary points.