MATH 364 – Midterm Exam Name: _____

1. (25 pts) Consider the following linear program.

$$\begin{array}{ll} max & z = 2x_1 - x_2 \\ s.t. & -x_1 + x_2 \geq 1 \\ & x_1 + x_2 \leq 4 \\ & x_2 \geq -1 \\ & \mathbf{x} \in \mathbb{R}^2 \end{array}$$

(a) Provide an accurate and labeled sketch of the feasible region.

(b) Solve this LP using the method of vertex enumeration. Clearly indicate your solution.

2. (25 pts) Consider the following nonlinear optimization problem:

$$\max f(x, y) = x + 2y \text{ subject to } y \leq 2x(2 - x).$$

(a) Provide an accurate labeled sketch of the feasible region and two or more iso-objective lines.

(b) Solve this problem by the *derivative-based method* learned in class. Justify your steps. Clearly indicate your solution.

3. (5 pts) I solved a linear program and obtained the results

$$x^* = \begin{bmatrix} 2.4 & 1.1 & -1.7 & 0.0 & -0.2 & 4.1 \end{bmatrix}^T \text{ and } z^* = 12.$$

Then I decided to restrict the decision variables to be integer. I obtained the new result

$$x^* = \begin{bmatrix} 2 & 1 & 0 & 0 & 3 & 3 \end{bmatrix}^T \text{ and } z^* = 14.$$

Is the problem a maximization or minimization problem? How do you know?

**For the following two modeling problems you are not required to justify your steps or reasoning. All that is required is a well-formulated, well-defined and clearly displayed integer program (IP).**

4. (25 pts) Furnco has recieved a government grant to manufacture chairs and desks for local schools. The cost of producing a chair is \$11, and a desk \$23. The grant stipulates that at least 200 total items must be made and there must be at least as many chairs as desks. Furthermore, at least 18 temporary jobs must be created. For every 20 chairs made, 1 temp job is created. For every 20 desks made, 3 temp jobs are created. Formulate an IP that can be used to minimize Furnco's cost.

5. (25 pts) I would like to pack some combination of items for shipment. My container can hold up to 160 pounds and has a capacity of 44 cubic feet. The list of items and their characteristics are given in the table below. Customs regulations stipulate that (a) the total combined weight of all vases and dishes cannot exceed half of the total shipped weight, and (b) at least one of each item must be shipped. Formulate an IP that can be used to maximize the total value of items shipped in this one container. Use a standard representation of your choice.

| # | item | in stock (quantity) | profit value (\$ per item) | weight (pounds per item) | volume ($ft^3$ per item) |
|---|---|---|---|---|---|
| 1 | vase | 5 | 98 | 11 | 2 |
| 2 | book | 12 | 57 | 4 | 1 |
| 3 | dish | 13 | 104 | 8 | 3 |
| 4 | shoe | 15 | 37 | 3 | 1 |