MATH 364 – Midterm Exam #2 Name: ____________________

- **Carefully read and follow the instructions for each problem!**
- You may use your personal notes or notes provided by me for the class, either written or electronic.
- You may **not** use any other sources.
- The maximum possible score on this exam is 110 out of 100.

1 [20]	2 [20]	3 [15]	4 [24]	5 [16]	6 [15]

1. [20 pts] A farmer would like to determine how many acres of wheat, lentils and garbanzos to plant in order to maximize total expected profit. The expected profit per acre for wheat is $40, for lentils $45 and for garbanzos $49. The farmer has 2300 acres of land. An inexplicable government regulation requires that the farmer plant at least as many acres in wheat as in lentils. In order to remain more robust to market instability, neither wheat nor lentils should exceed half of the total planted acreage, and garbanzos should not exceed 1500 total acres. The farmer also has the option to rent and plant 700 acres of neighboring land at a fixed cost of $32,000.

   Construct a *matrix form* mixed-integer program which can be used to determine the farmer's planting strategy for maximum profit. You do not need to show your work. However, you must provide a clearly identified MIP and variable definitions.

2. [20 pts] A food manufacturer produces corn syrup, fig paste, and fig bars. There is unlimited demand for these products at the respective selling prices per unit of \$0.25, \$1.20, and \$3.50. Producing one unit of corn syrup requires 1/2 hour of labor. Producing one unit of fig paste requires 1.5 hours of labor *and* one unit of corn syrup. Producing one unit of fig bars requires 2 hours of labor *and* two units of fig paste. 300 hours of labor are available.

   Construct a *matrix form* integer program whose solution determines how revenue can be maximized. You do not need to show your work. However, you must provide a clearly identified IP and variable definitions.

3. [15 pts] Provide linear constraints which model the given constraint conditions. You do not need to explain, but you must **draw a box around your answers**.

   (a) $(a \leq x \leq b)$ or $(c \leq x \leq d)$, where a, b, c, d are given constants.

   (b) (If $y = -1$ then $x = 1$) and (if $y = 1$ then $x = 0$), where $y \in \{-1, 0, 1\}$ and $x \in \{0, 1\}$.

   (c) $|x + y - 8| \leq 3$, with $x, y \in \mathbb{R}$.

4. Consider the following Simplex Tableau derived from a maximization problem.

$$\begin{array}{rcrcrcrcrcr} x_3 & = & 12 & + & 4x_4 & - & 2x_6 & - & 3x_2 & + & 2x_5 \\ x_1 & = & 3 & - & 6x_4 & - & 2x_6 & - & 3x_2 & - & x_5 \\ \hline z & = & -4 & + & 5x_4 & + & x_6 & - & 3x_2 & + & x_5 \end{array}$$

Answer the following questions **with justification**.

(a) [2 pts] What is the current basis?

(b) [2 pts] What is the current basic feasible solution?

(c) [3 pts] List the entering and exiting variables for each valid pivot.

(d) [3 pts] Which pivot would provide the largest increase in objective value? (You do NOT need to perform each pivot to find the answer.)

(e) [15 pts] Starting with the pivot form part (c), complete the Simplex Method solution. (Use the back of this page if necessary.)

5. Consider the following Simplex Tableau derived from a maximization problem, where $a, b, c, d, e, f \in \mathbb{R}$.

$$\begin{array}{rcrcrcrcr} x_3 & = & 3 & + & bx_4 & + & ex_6 & - & 3x_2 \\ x_1 & = & a & + & cx_4 & - & 2x_6 & + & fx_2 \\ \hline z & = & -4 & + & dx_4 & + & 2x_6 & - & 3x_2 \end{array}$$

Provide values for a, b, c, d, e, f which correspond to the stated condition:

(a) [4 pts] The LP is unbounded.

(b) [4 pts] The current basic feasible solution is degenerate.

(c) [4 pts] There is exactly one valid pivot.

(d) [4 pts] The current basic solution is infeasible.

6. Consider the following linear program, where a_k, b_k are given constants.

$$\begin{aligned} \min_{x,y} \quad & y \\ s.t. \quad & a_k x + b_k \le y, \quad k = 1, 2, \cdots, m \\ & x, y \in \mathbb{R} \end{aligned}$$

Answer the following questions.

(a) [2 pts] How many decision variables are there?

(b) [2 pts] How many constraints are there?

(c) [3 pts] Construct the corresponding matrix form LP.

(d) [4 pts] Describe what this LP models.

(e) [4 pts] Sketch an example situation for $m = 4$ and indicate the optimal point on your sketch.