MATH 364 – Midterm Exam Name: ______________________________

Show your work. You must briefly explain what you are doing. Use correct linear algebra and optimization language. Complete all problems.

1. [20 pts] Using the methods presented in class, solve the following nonlinear optimization problem.

$$\begin{aligned} \max \quad f(x,y) = \; & y^2 + xe^{-x^2} \\ s.t. \quad & -2 \leq x \leq 2 \\ & x, y \in \mathbb{R} \end{aligned}$$

2. [20 pts] Reformulate the following linear program to be in (equality) standard form. Write your result in matrix form. Do not solve.

$$\begin{aligned} \max \quad z = \; & x_1 + 4x_2 + 4x_3 \\ s.t. \quad & x_1 + 3x_2 + x_3 \leq 12 \\ & x_1 + 2x_2 + x_3 \geq 4 \\ & x_1 + x_2 = 3 \\ & x \geq 0 \\ & x \in \mathbb{R}^3 \end{aligned}$$

3. [20 pts] Solve the following linear program using a vertex enumeration method. You may assume that the feasible region is bounded.

$$\begin{aligned} \min \quad z = \; & 2x_1 + x_2 \\ s.t. \quad & x_1 + x_2 \geq -1 \\ & x_1 - x_2 \leq 5 \\ & x_1 \geq 0 \\ & x_2 \leq 0 \\ & x \in \mathbb{R}^2 \end{aligned}$$

4. [20 pts] Plot the $z = 0$ and $z = \pm 1$ level curves of the function $z = f(x, y) = (x - 1)(y + 2)$. Show that the only stationary point is a saddle point.

5. [10 pts] Is the following set of constraints linearly independent or linearly dependent?

$$\begin{aligned} x_1 + x_2 + x_3 &= 3 \\ x_2 + x_3 &\leq 2 \\ x_3 &\geq 0 \end{aligned}$$

6. [10 pts] Carefully argue and show that the following constraint set can be used in a linear program.

$$\frac{x_1 + 2x_2}{1 + x_1 + x_2} \geq 5 \quad \text{and} \quad x_1 + x_2 \geq 0$$