(24) Chemco produces two chemicals: A and B. These chemicals are produces via two manufacturing processes. Process 1 requires 2 hours of labor and 15 pounds of raw material to produce 2 pounds of A and 1 pound of B. Process 2 requires 3 hours of labor and 28 pounds of raw material to produce 3 pounds of A and 2 pounds of B. Sixty hours of labor and 600 pounds of raw material are available. Demand for A is unlimited, but only 20 pounds of B can be sold. A sells for \$16 per pound, and B sells for \$13 per pound. Any B that is unsold must be disposed of at a cost of \$2 per pound. Formulate a LP to maximize Chemco's profit. Use appropriate software to find solutions. Describe your solution in terms of the problem description.

PROBLEM 24

(1) Decision Variables. I will use 3 decision variables:

X_1 = # of times to run process 1 $\quad X_1 \in \mathbb{Z}$

X_2 = # of times to run process 2 $\quad X_2 \in \mathbb{Z}$

X_3 = pounds of excess chemical B to be disposed of. $\quad X_3 \in \mathbb{R}$

The last variable is not independent, so we will need to constrain it (later).

(2) Objective Function. Maximize profit

Z = (profit \$)

Z = (chem A profit) + (chem B profit) − (disposal cost)

$Z = 16(2X_1 + 3X_2) + 14(X_1 + 2X_2 - X_3) - 2X_3$

$Z = 46X_1 + 76X_2 - 16X_3$

(3) Constraints. We have limited labor and raw materials

$2X_1 + 3X_2 \leq 60$

$2X_1$: process 1 labor hours; 60: total labor available

$16X_1 + 32X_2 \leq 640$

$16X_1$: process 1 raw materials; 640: total raw materials

We also must ensure that the excess X_3 is measured in terms of X_1 and X_2:

(pounds B produced) ≤ (20 pounds) + (excess pounds)

$X_1 + 2X_2 - X_3 \leq 20$

(4) Sign Restrictions, $x \geq 0$ (production & excess non-negative)

(5) Formulate the MIP:

$$\max\ z = c^T x \qquad c = [46 \quad 76 \quad -16]$$

$$\text{s.t}\ \ Ax \le b$$

$$x \ge 0$$

$$x_1, x_2 \in \mathbb{Z}$$

$$x_3 \in \mathbb{R}$$

$$A = \begin{bmatrix} 2 & 3 & 0 \\ 16 & 32 & 0 \\ 1 & 2 & -1 \end{bmatrix} \qquad b = \begin{bmatrix} 60 \\ 640 \\ 20 \end{bmatrix}$$

Solution by Octave:

$$x^* = \begin{bmatrix} 30 \\ 0 \\ 10 \end{bmatrix} \qquad z^* = 1220$$

The maximum profit is \$1220. To acheive this, run process 1 30 times, process 2 not at all, pay to dispose of 10 pounds of chemical B.