

Samples of Good Solutions to Homework Problems

Suppose the current course topic is the implementation of the product rule for differentiation. Provide a solution to the following homework problem.

1. Differentiate the following functions.

(a) $f(x) = 3x^2 \sin x$.

(b) $f(x) = \ln x \cos x$.

Solution: The product rule for differentiation states that for a function $f(x) = g(x)h(x)$, $f'(x) = g'(x)h(x) + g(x)h'(x)$.

(a) $f(x) = 3x^2 \sin x$.

Let $g(x) = 3x^2$ and $h(x) = \sin x$, then

$$\begin{aligned} f'(x) &= (6x)(\sin x) + (3x^2)(\cos x) \\ &= 3x(2 \sin x + x \cos x). \end{aligned}$$

(b) $f(x) = \ln x \cos x$.

Let $g(x) = \ln x$ and $h(x) = \cos x$, then

$$\begin{aligned} f'(x) &= (1/x)(\cos x) + (\ln x)(-\sin x) \\ &= \frac{\cos x}{x} - \ln x \sin x. \end{aligned}$$

Suppose the current course topic is modeling and solving word problems using algebra. Provide a solution to the following homework problem.

1. The railroad track from Chicago to Minneapolis is 400 miles long. The SteamKing train leaves Chicago at noon heading for Minneapolis and the IceBreaker train leaves Minneapolis at noon heading for Chicago. The SteamKing travels at 70 miles per hour and the IceBreaker travels at 55 miles per hour. Answer the following questions:
 - (a) At what time do the two trains pass each other?
 - (b) At what location along the track do the two trains pass each other?

Solution Let x be the distance along the track from Chicago (mile zero) to Minneapolis (mile 400). Let t be the time in hours from noon. The location of the SteamKing is

$$x_{sk}(t) = 70t \tag{1}$$

and the location of the IceBreaker is

$$x_{ib}(t) = 400 - 55t. \tag{2}$$

The trains meet after a time t when $x_{sk} = x_{ib}$.

$$x_{ib} = x_{sk} \tag{3}$$

$$400 - 55t = 70t \tag{4}$$

$$400 = 15t \tag{5}$$

$$\frac{8}{3} = t \tag{6}$$

We see that the time the two trains meet is $\frac{8}{3}$ hours past noon, or 2:40 PM. We can use the travel time (Equation 6) and either of Equations 1 or 2 to determine the location x where the trains meet.

$$x = 70t = (70)(8/3) = \frac{560}{3}. \tag{7}$$

That is, the trains meet $560/3 \approx 187$ miles from Chicago.