

3. You own a company that assembles computers for retail stores. The production costs per computer during the next four months are \$250, \$400, \$200, and \$350, respectively. The company can make no more than 85 computers in any month. Your customer requires 50, 65, 100 and 70 computers over the next four months. Your company can store completed computers from one month to the next at a cost of \$20 per computer, but no more than 20 computers can be stored at any time. You estimate that any computers left over after month 4 can be sold for \$500 each in the facility showroom. Formulate and solve an IP that determines a lowest-cost production strategy that meets consumer demand.

Let X_k ($k=1,2,3,4$) be the number of computers to assemble in month k .
 Let S_k ($k=1,2,3,4$) be the number of computers to store after month k .

The objective is to reduce (minimize) the total business cost over the entire year: let Z be the cost in (\$).

$$Z = \underbrace{250X_1 + 400X_2 + 200X_3 + 350X_4}_{\text{production cost}} + \underbrace{20 \sum_{k=1}^4 S_k}_{\text{storage cost}} - \underbrace{500 S_4}_{\text{selling profit}}$$

Limits on production and storage:

$$0 \leq X_k \leq 85$$

$$0 \leq S_k \leq 20$$

production and storage are related through sales:

$$X_1 - 50 = S_1$$

$$S_1 + X_2 - 65 = S_2 \quad \leftarrow \text{carry over from month 1 plus production from month 2 minus month 2 sales equals what must be stored after month 2.}$$

$$S_2 + X_3 - 100 = S_3$$

$$S_3 + X_4 - 70 = S_4$$

~~min z = c^T x~~

$$\min z = c^T x$$

$$x \in \mathbb{Z}^8$$

$$\text{s.t. } Ax = b$$

$$l \leq x \leq u$$

$$x = [x_1 \ x_2 \ x_3 \ x_4 \ s_1 \ s_2 \ s_3 \ s_4]^T$$

$$c = [250 \ 400 \ 200 \ 350 \ 20 \ 20 \ 20 \ -480]^T$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 50 \\ 65 \\ 100 \\ 70 \end{bmatrix}$$

$$l = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$$

$$u = [85 \ 85 \ 85 \ 85 \ 20 \ 20 \ 20 \ 20]^T$$