Calculus and Optimization II

In our class discussion we re-invented Newton's Method for finding the roots of a known function $f(x)$ of one variable. We found that, given an initial guess for a root x_0, we find a sequence of approximations

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}, \quad k = 0, 1, 2, \ldots$$

which hopefully converge to a root of f. We developed this method by considering a local linear approximation to the function. In particular, we observed:

$$f(x + a) \approx f(x) + af'(x).$$

1. Building on our work, develop a similar method for finding the *stationary points* of a function of one variable $f(x)$.
2. Use your new iterative procedure to find a stationary point of $f(x) = e^x - x - 2$ using the starting guess $x_0 = 1$.

(1) To find a stationary point algorithm to find x such that $f'(x) = 0$, we note that $f'(x)$ is a function for which the given method can be applied. We have

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}, \quad k = 0, 1, 2, \cdots$$

We can choose to terminate the iteration when either $|x_{k+1} - x_k|$ or $|f'(x_k)|$ is sufficiently small.

(2) For $f(x) = e^x - x - 2$, we have $f'(x) = e^x - 1$, $f''(x) = e^x$, and the procedure:

$$x_{k+1} = x_k - \frac{e^{x_k} - 1}{e^{x_k}} = x_k - 1 + e^{-x_k}.$$

We have the starting point $x_0 = 1$ and find the sequence of approximations

$$x_0 = 1, \quad x_1 = 0.3679, \quad x_2 = 0.06008, \quad x_3 = 1.769 \times 10^{-3}$$

HOMEWORK: make a table of values for x_k, $f(x_k)$, $f'(x_k)$, $f''(x_k)$ for $k = 0, 1, 2, 3$ and comment on the sucess of the method.