Math 364 – In Class Assignment #3 – Solutions

Quiz (12 minutes)

Let $V = \{v_1, v_2, \cdots, v_k\}$ be a set of vectors from $\mathbb{R}^n$. How might you determine if the set V is linearly independent or linearly dependent? List as many ways as you can. You must use correct linear algebra language.

The following list of tests is not necessarily comprehensive and some tests are repeated with different wording. Let A be the $n \times k$ matrix whose columns are the vectors $v_1, v_2, \cdots, v_k$.

1. Basic tests
   (a) If $v_i = av_j$ for some scalar a and $i \neq j$, then V is L.D.
   (b) If $v_i = 0$ for some $0 \leq i \leq k$, then V is L.D.
   (c) If $k > n$ then V is L.D.
   (d) If $V = \emptyset$, then V is L.I.
   (e) If any vector in V can be written as a linear combination of the other vectors in V, then V is L.D.
2. Suppose $k = n$. If any of the following statements are true, then V is L.I., otherwise V is L.D.
   (a) $\det(A) \neq 0$.
   (b) A is row equivalent to I_n.
   (c) $Ax = 0$ has only the trivial solution.
   (d) A is invertible.
3. Suppose $k \leq n$. If any of the following statements are true, then V is L.I., otherwise V is L.D.
   (a) $a_1v_1 + a_2v_2 + \cdots + a_kv_k = 0$ has only the trivial solution $a_1 = a_2 = \cdots a_k = 0$.
   (b) $a_1v_1 + a_2v_2 + \cdots + a_kv_k = b$ has a unique solution for all $b \in \mathbb{R}$.
   (c) $\dim \operatorname{span}(V) = k$.
   (d) There does not exist a proper subset $S \subset V$ such that $\operatorname{span}(S) = \operatorname{span}(V)$.

(e) A is injective (one-to-one).

(f) V is a basis for a k-dimensional subspace of $\mathbb{R}^n$.

(g) A has rank k.

4. Other tests.

    (a) If $k = n$, A is diagonalizable and all eigenvalues of A are nonzero, then V is L.I.

    (b) If $k = n$ and A has at least one zero eigenvalue, then V is L.D.

    (c) If the reduced row echelon form of A has a zero row, then V is L.D., otherwise V is L.I.