Math 364 – Homework #2

Due: Tuesday September 5, in class

1. Briefly explain why the given optimization problems are not linear programs.

   (a)

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & f(x) = (x_1 - 2)^2 + (x_2 - 1)^2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 0 \end{aligned}$$

   (b)

$$\begin{aligned} \min_{x \in \mathbb{Z}^2} \quad & f(x) = 10x_1 - 33x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 0 \\ & x_1 + 5x_2 = 7 \end{aligned}$$

   (c)

$$\begin{aligned} \max_{x \in \mathbb{R}^2} \quad & f(x) = 2x_1 - 3x_2 \\ \text{s.t.} \quad & x_1 (1 + x_2) \leq 5 \\ & 2x_1 + 3x_2 \geq -4 \\ & x_1 - x_2 = 0 \end{aligned}$$

2. Show, by using sketches of several representative Linear Programs in two dimensions, that if an optimal solution exists for a LP, then at least one vertex of the feasible region is optimal. Sketches should include feasible regions and level curves of objective funcions. Provide a convincing argument. In words, provide a justification that the same conclusion is true in higher dimensions.

3. Graphically, we can solve a linear program by checking each vertex of the feasible region. Provide a similar solution method for linear programs where we cannot determine all of the vertices directly. (This can occur if the problem is high-dimensional and cannot be drawn, or has many constraints and would be too time consuming to sketch.)