# MATH 448/548 - Numerical Analysis

**Final Project (for undergrad. students)**
*Date assigned:* April 8, 2008
*Due date:* **May 2, 2008**

- Include a cover page and *this* problem sheet
- Include the printout of your program(s) for completeness

Consider the following diffusion equation:

$$\frac{\partial v}{\partial t} = D\frac{\partial^2 v}{\partial x^2} \ = 0,\ x \in [0,1],\ t > 0, \tag{1}$$

with initial and boundary conditions given by

$$v(x,0) = \begin{cases} x, & \text{if } 0 \le x \le 0.5, \\ 1-x, & \text{if } 0.5 < x \le 1. \end{cases} \tag{2}$$

$$v(0,t) = v(1,t) = 0,\ t >= 0. \tag{3}$$

Numerically solve (1)-(3) with the diffusion coefficient $D = 1/2$.
Use the following difference schemes:

(ES) explicit

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = D\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}$$

(IS) implicit

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = D\frac{u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}}{\Delta x^2}$$

Take $\Delta x = 0.1$ and compute the numerical solution of (1)-(3) with $\Delta t = 1/50, 1/100, 1/200$.

**Questions.**

- What are stability conditions and order of accuracy for both numerical schemes?
- Numerically check the order of accuracy in time by showing the error between exact and numerical solution for different values of $\Delta t$.
- Compare plots of the exact and numerical solution for each scheme at $\Delta t = 1/50, 1/100, 1/200$.
- Explain the results of the computations.