# MATH 448/548 - Numerical Analysis

**Final Project (for grad. students)**
*Date assigned:* April 8, 2008
*Due date:* **May 2, 2008**

- Include a cover page and *this* problem sheet
- Include the printout of your program(s) for completeness

Consider the following advection equation:

$$\frac{\partial v}{\partial t} + a\frac{\partial v}{\partial x} = 0,\ x \in [0, 15],\ t >= 0, \tag{1}$$

with initial and boundary conditions given by

$$v(x, 0) = e^{-0.8(x-5)^2} + e^{-8(x-9)^2},\ x \in [0, 15], \tag{2}$$

$$v(0, t) = v(15, t),\ t >= 0. \tag{3}$$

Numerically solve (1)-(3) considering two cases for the speed of propagation:

- a=2,
- a=-0.5.

Use the following difference schemes:

1. One-sided FTBS scheme .
2. One-sided FTFS scheme .
3. Explicit Leapfrog scheme.

$$\frac{u_j^{n+1} - u_j^{n-1}}{2\Delta t} = -a\frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x}.$$

4. Explicit Lax-Wendroff scheme.

5. Implicit Centered scheme.

Take $\Delta x = 0.05$ and appropriate $\Delta t$.

For each scheme:

- Show the order of accuracy in time and space.
- Show stability condition.
- Compare plots of the initial condition and numerical solution at $t = 20$.

Answer the following questions:

1. Will the quality of the solution improve if you take $\Delta x < 0.05$? Why?
2. Explain the results of your computations. Which scheme gives better solution and why?